Glencoe Mathematics

Algebra 1

Chapter 9
Resource Masters

Includes:

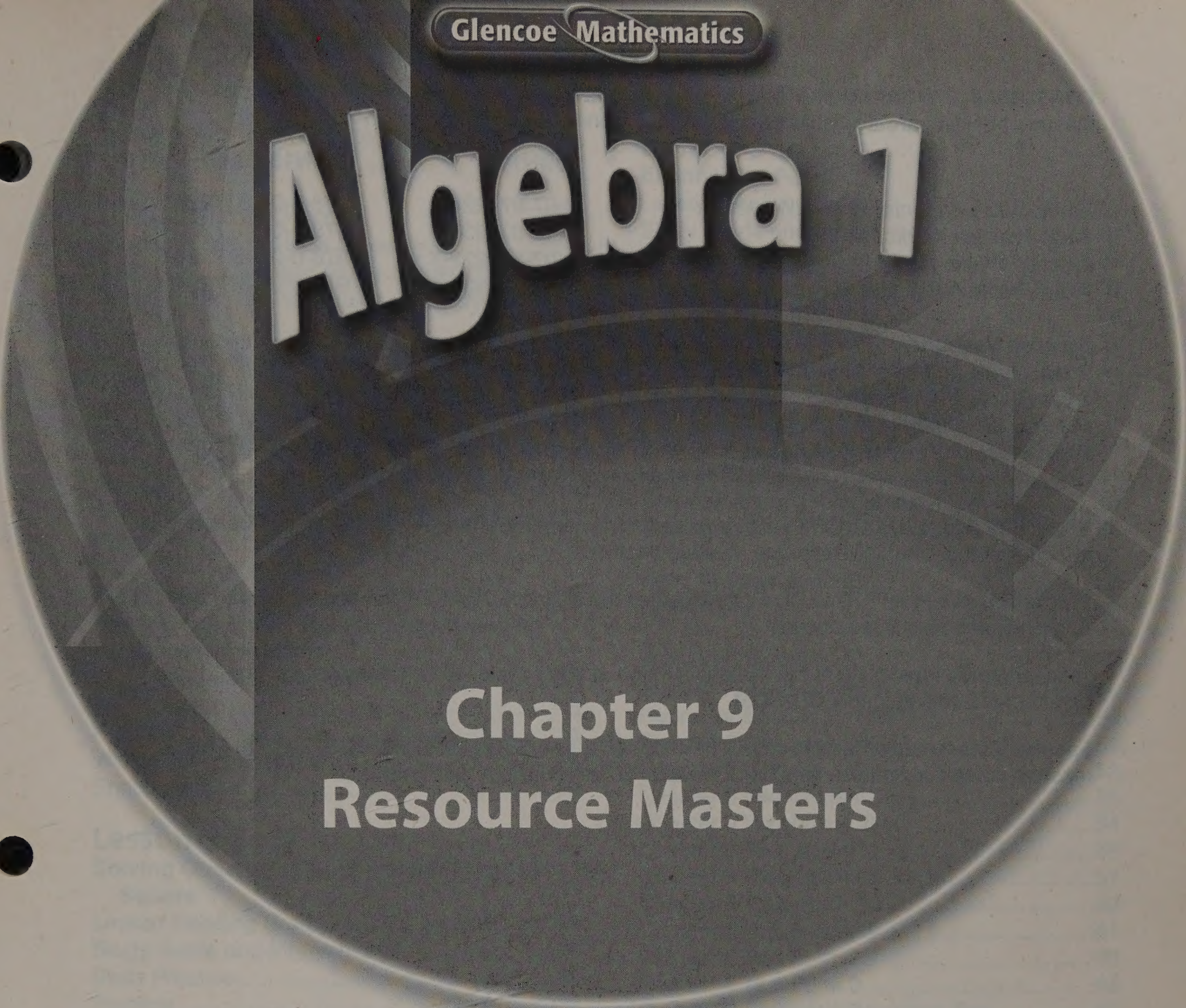
Chapter Resources
- Student-Built Glossary
- Anticipation Guide

Leveled Lesson Resources
- Lesson Reading Guide
- Study Guide and Intervention
- Skills Practice
- Practice
- Word Problem Practice
- Enrichment

Technology Resources
- TI-83/84 Plus Activity
- Spreadsheet Activity

Assessment Resources
- Practice Answer Sheet for Standardized Test Practice
- 4 Quizzes
- Mid-Chapter Test
- Vocabulary Test
- Leveled Chapter 9 Tests
- Extended-Response Test
- Standardized Test Practice
- Unit 3 Test

All Answers Included



Glencoe Mathematics

Algebra 1

Chapter 9
Resource Masters

Glencoe

New York, New York   Columbus, Ohio   Chicago, Illinois   Peoria, Illinois   Woodland Hills, California

CONSUMABLE WORKBOOKS Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks in both English and Spanish.

	ISBN10	ISBN13
Study Guide and Intervention Workbook	0-07-877228-1	978-0-07-877228-3
Skills Practice Workbook	0-07-877230-3	978-0-07-877230-6
Practice Workbook	0-07-877232-X	978-0-07-877232-0
Word Problem Practice Workbook	0-07-877234-6	978-0-07-877234-4
Spanish Versions		
Study Guide and Intervention Workbook	0-07-877229-X	978-0-07-877229-0
Skills Practice Workbook	0-07-877231-1	978-0-07-877231-3
Practice Workbook	0-07-877233-8	978-0-07-877233-7
Word Problem Practice Workbook	0-07-877235-4	978-0-07-877235-1

ANSWERS FOR WORKBOOKS The answers for Chapter 9 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

StudentWorks Plus™ This CD-ROM includes the entire Student Edition text along with the English workbooks listed above.

TeacherWorks Plus™ All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

Spanish Assessment Masters (ISBN10: 0-07-877236-2, ISBN13: 978-0-07-877236-8) These masters contain a Spanish version of Chapter 9 Test Form 2A and Form 2C.

Glencoe

The McGraw-Hill Companies

Copyright © by The McGraw-Hill Companies, Inc. All rights reserved.
Permission is granted to reproduce the material contained herein on the condition that such material be reproduced only for classroom use; be provided to students, teachers, and families without charge; and be used solely in conjunction with Glencoe *Algebra 1*. Any other reproduction, for use or sale, is prohibited without prior written permission of the publisher.

Send all inquiries to:
The McGraw-Hill Companies
8787 Orion Place
Columbus, OH 43240

ISBN13: 978-0-07-873952-1
ISBN10: 0-07-873952-7

Algebra 1 CRM9

Printed in the United States of America

2 3 4 5 6 7 8 9 10 024 13 12 11 10 09 08 07

Contents

Teacher's Guide to Using the Chapter 9 Resource Masters iv

Chapter Resources

Chapter 9 Student-Built Glossary 1
Chapter 9 Anticipation Guide (English).............3
Chapter 9 Anticipation Guide (Spanish) 4

Lesson 9-1

Graphing Quadratic Functions

Lesson Reading Guide 5
Study Guide and Intervention 6
Skills Practice...8
Practice 9
Word Problem Practice 10
Enrichment..11

Lesson 9-2

Solving Quadratic Equations by Graphing

Lesson Reading Guide 12
Study Guide and Intervention 13
Skills Practice...15
Practice 16
Word Problem Practice 17
Enrichment..18

Lesson 9-3

Solving Quadratic Equations by Completing the Square

Lesson Reading Guide 19
Study Guide and Intervention 20
Skills Practice...22
Practice 23
Word Problem Practice 24
Enrichment..25

Lesson 9-4

Solving Quadratic Equations by Using the Quadratic Formula

Lesson Reading Guide 26
Study Guide and Intervention 27
Skills Practice...29
Practice 30
Word Problem Practice 31
Enrichment..32

Lesson 9-5

Exponential Functions

Lesson Reading Guide 33
Study Guide and Intervention 34
Skills Practice...36
Practice 37
Word Problem Practice 38
Enrichment..39
Spreadsheet...40

Lesson 9-6

Growth and Decay

Lesson Reading Guide 41
Study Guide and Intervention 42
Skills Practice...44
Practice 45
Word Problem Practice 46
Enrichment..47

Assessment

Student Recording Sheet 49
Rubric for Scoring Pre-AP...............................50
Chapter 9 Quizzes 1 and 2............................51
Chapter 9 Quizzes 3 and 4............................52
Chapter 9 Mid-Chapter Test...........................53
Chapter 9 Vocabulary Test 54
Chapter 9 Test, Form 1 55
Chapter 9 Test, Form 2A...............................57
Chapter 9 Test, Form 2B...............................59
Chapter 9 Test Form 2C................................61
Chapter 9 Test Form 2D................................63
Chapter 9 Test Form 3 65
Chapter 9 Extended-Response Test 67
Standardized Test Practice.............................68
Unit 3 Test 71

Answers.......................................A1-A33

Teacher's Guide to Using the *Chapter 9 Resource Masters*

The *Chapter 9 Resource Masters* includes the core materials needed for Chapter 9. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the *TeacherWorks Plus™* CD-ROM.

Chapter Resources

Student-Built Glossary (pages 1-2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 9-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

Anticipation Guide (pages 3-4) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

Lesson Reading Guide Get Ready for the Lesson extends the discussion from the beginning of the Student Edition lesson. Read the Lesson asks students to interpret the context of and relationships among terms in the lesson. Finally, Remember What You Learned asks students to summarize what they have learned using various representation techniques. Use as a study tool for note taking or as an informal reading assignment. It is also a helpful tool for ELL (English Language Learners).

Study Guide and Intervention These masters provide vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

Practice This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.

Word Problem Practice This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

Enrichment These activities may extend the concepts of the lesson, offer a historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. They are written for use with all levels of students.

Graphing Calculator, Scientific Calculator, or Spreadsheet Activities These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

Assessment Options

The assessment masters in the ***Chapter 9 Resource Masters*** offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

Student Recording Sheet This master corresponds with the standardized test practice at the end of the chapter.

Pre-AP Rubric This master provides information for teachers and students on how to assess performance on open-ended questions.

Quizzes Four free-response quizzes offer assessment at appropriate intervals in the chapter.

Mid-Chapter Test This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

Vocabulary Test This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students' knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

Leveled Chapter Tests

- ***Form 1*** contains multiple-choice questions and is intended for use with below grade level students.
- ***Forms 2A and 2B*** contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- ***Forms 2C and 2D*** contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- ***Form 3*** is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

Extended-Response Test Performance assessment tasks are suitable for all students. Samples answers and a scoring rubric are included for evaluation.

Standardized Test Practice These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

Answers

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters.

NAME ______________________________ DATE ____________ PERIOD _____

9 Student-Built Glossary

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 9. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Algebra Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
axis of symmetry SIH·muh·tree		
completing the square		
compound interest		
discriminant		
double root		
exponential (EHK·spuh·NEHN·chuhl) decay		
exponential function		
exponential growth		
maximum		
minimum		

(continued on the next page)

9 Student-Built Glossary *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
parabola puh·RA·buh·luh		
quadratic (kwah·DRA·tihk) equation		
Quadratic Formula		
quadratic function		
roots		
symmetry		
vertex		
zeros		

NAME ______________________________ DATE ____________ PERIOD _____

9 Anticipation Guide

Quadratic and Exponential Functions

Step 1 ***Before you begin Chapter 9***

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. The graph of a quadratic function is a parabola.	
	2. The graph of $4x^2 - 2x + 7$ will be a parabola opening downward since the coefficient of x^2 is positive.	
	3. A quadratic function's axis of symmetry is either the x-axis or the y-axis.	
	4. The graph of a quadratic function opening upward has no maximum value.	
	5. The x-intercepts of the graph of a quadratic function are the solutions to the related quadratic equation.	
	6. All quadratic equations have two real solutions.	
	7. Any quadratic expression can be written as a perfect square by a method called *completing the square.*	
	8. The quadratic formula can only be used to solve quadratic equations that cannot be solved by factoring or graphing.	
	9. A function containing powers is called an exponential function.	
	10. Receiving compound interest on a bank account is one example of exponential growth.	

Step 2 ***After you complete Chapter 9***

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NOMBRE ______________________ FECHA ____________ PERÍODO ____

9

Ejercicios preparatorios

Funciones cuadráticas y exponenciales

Paso 1 *Antes de comenzar el Capítulo 9*

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estoy seguro(a).

PASO 1 A, D o NS	Enunciado	PASO 2 A o D
	1. La gráfica de una función cuadrática es una parábola.	
	2. La gráfica de $4x^2 - 2x + 7$ será una parábola que se abre hacia abajo, puesto que el coeficiente de x^2 es positivo.	
	3. El eje de simetría de una función cuadrática es el eje x o el eje y.	
	4. La gráfica de una función cuadrática que se abre hacia arriba no tiene un valor máximo.	
	5. Las intersecciones x de la gráfica de una función cuadrática son las soluciones de la ecuación cuadrática relacionada.	
	6. Todas las ecuaciones cuadráticas tienen dos soluciones reales.	
	7. Cualquier expresión cuadrática puede escribirse como un cuadrado perfecto mediante el método denominado *completar el cuadrado.*	
	8. La fórmula cuadrática sólo puede usarse para resolver ecuaciones cuadráticas que no pueden resolverse mediante factorización o gráficas.	
	9. Una función que contiene potencias se denomina función exponencial.	
	10. El recibir interés compuesto en una cuenta bancaria es un ejemplo de crecimiento exponencial.	

Paso 2 *Después de completar el Capítulo 9*

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.

NAME ______________________________ DATE ____________ PERIOD _____

9-1 Lesson Reading Guide

Graphing Quadratic Functions

Get Ready for the Lesson

Read the introduction to Lesson 9-1 in your textbook.

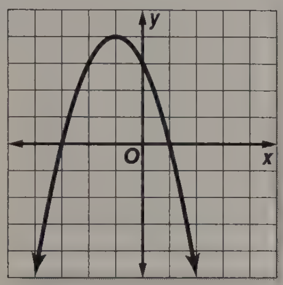
According to the graph, at what height does the rocket explode and in how many seconds after being launched?

Read the Lesson

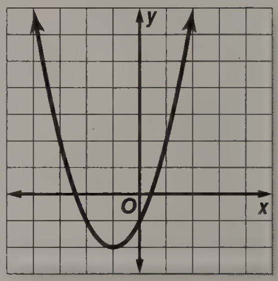
1. The standard form for a ______________ function is $y = ax^2 + bx + c$. For the function $y = 2x^2 - 5x + 3$, the value of a is ________, the value of b is ________, and the value of c is ________.

2. The graphs of two quadratic functions are shown below. Complete each statement about the graphs.

A.



B.



a. Each graph is a curve called a ______________.

b. The highest point of graph A is located at ______________. This point is the ______________ (maximum/minimum) point of the graph.

c. The lowest point of graph B is located at ______________. This point is the ______________ (maximum/minimum) point of the graph.

3. The maximum or minimum point of a parabola is called the ______________ of the parabola.

4. If you fold a parabola along a line to get two halves that match exactly, the line where you fold the parabola is the ______________________ of the parabola. This line goes through the ______________ of the parabola.

5. For a quadratic function $y = ax^2 + bx + c$, the parabola opens upward if a ________ 0. It opens downward if a ________ 0.

Remember What You Learned

6. Look up the word *vertex* in a dictionary. You will find that it comes from the Latin word *vertere*, which means to turn. How can you use the idea of "to turn" to remember what the vertex of a parabola is?

NAME ______________________________ DATE ____________ PERIOD _____

9-1 Study Guide and Intervention

Graphing Quadratic Functions

Graph Quadratic Functions

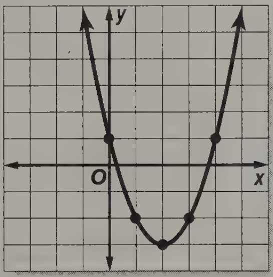
Quadratic Function	a function described by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$	**Example:** $y = 2x^2 + 3x + 8$

The parent graph of the family of quadratic fuctions is $y = x^2$. Graphs of quadratic functions have a general shape called a **parabola**. A parabola opens upward and has a **minimum point** when the value of a is positive, and a parabola opens downward and has a **maximum point** when the value of a is negative.

Example 1

a. Use a table of values to graph $y = x^2 - 4x + 1$.

x	y
−1	6
0	1
1	−2
2	−3
3	−2
4	1



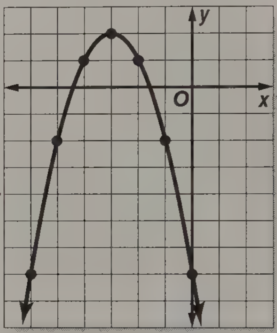
Graph the ordered pairs in the table and connect them with a smooth curve.

b. What is the domain and range of this function?

Example 2

a. Use a table of values to graph $y = -x^2 - 6x - 7$.

x	y
−6	−7
−5	−2
−4	1
−3	2
−2	1
−1	−2
0	−7



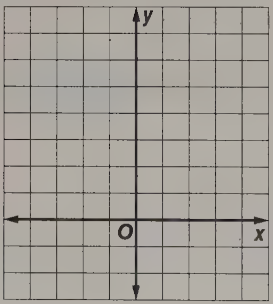
Graph the ordered pairs in the table and connect them with a smooth curve.

b. What is the domain and range of this function?

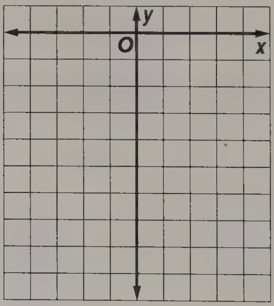
Exercises

Use a table of values to graph each function. Determine the domain and range.

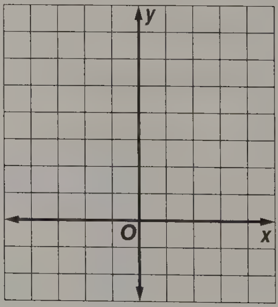
1. $y = x^2 + 2$



2. $y = -x^2 - 4$



3. $y = x^2 - 3x + 2$



NAME ______________________________ DATE ______________ PERIOD _____

9-1 Study Guide and Intervention *(continued)*

Graphing Quadratic Functions

Symmetry and Vertices Parabolas have a geometric property called **symmetry**. That is, if the figure is folded in half, each half will match the other half exactly. The vertical line containing the fold line is called the **axis of symmetry**.

Axis of Symmetry	For the parabola $y = ax^2 + bx + c$, where $a \neq 0$, the line $x = -\frac{b}{2a}$ is the axis of symmetry.	**Example:** The axis of symmetry of $y = x^2 + 2x + 5$ is the line $x = -1$.

The axis of symmetry contains the minimum or maximum point of the parabola, the **vertex**.

Example **Consider the graph of $y = 2x^2 + 4x + 1$.**

a. Write the equation of the axis of symmetry.

In $y = 2x^2 + 4x + 1$, $a = 2$ and $b = 4$. Substitute these values into the equation of the axis of symmetry.

$$x = -\frac{b}{2a}$$

$$x = -\frac{4}{2(2)} = -1$$

The axis of symmetry is $x = -1$.

b. Find the coordinates of the vertex.

Since the equation of the axis of symmetry is $x = -1$ and the vertex lies on the axis, the x-coordinate of the vertex is -1.

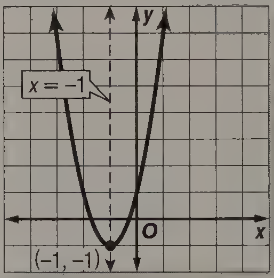
$y = 2x^2 + 4x + 1$	Original equation
$y = 2(-1)^2 + 4(-1) + 1$	Substitute.
$y = 2(1) - 4 + 1$	Simplify.
$y = -1$	

The vertex is at $(-1, -1)$.

c. Identify the vertex as a maximum or a minimum.

Since the coefficient of the x^2-term is positive, the parabola opens upward, and the vertex is a minimum point.

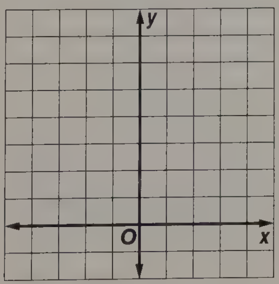
d. Graph the function.



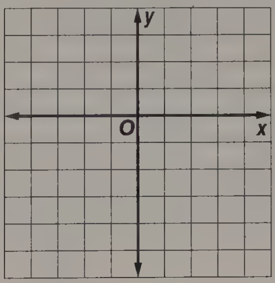
Exercises

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or a minimum. Then graph the function.

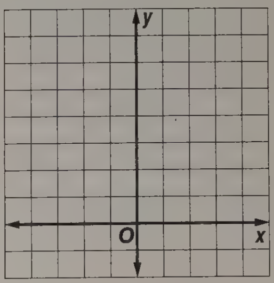
1. $y = x^2 + 3$



2. $y = -x^2 - 4x - 4$



3. $y = x^2 + 2x + 3$



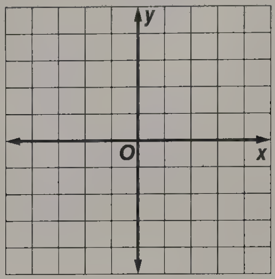
NAME ______________________ DATE ____________ PERIOD _____

9-1 Skills Practice

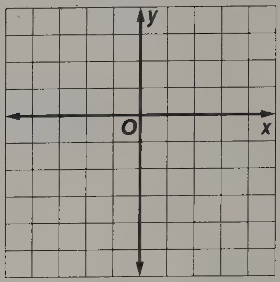
Graphing Quadratic Functions

Use a table of values to graph each function.

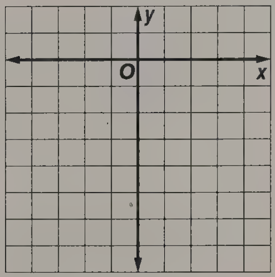
1. $y = x^2 - 4$



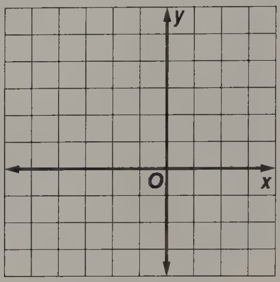
2. $y = -x^2 + 3$



3. $y = x^2 - 2x - 6$

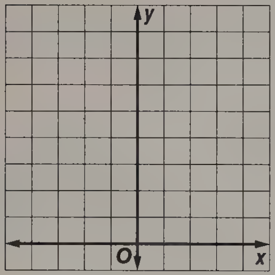


4. $y = -x^2 - 4x + 1$

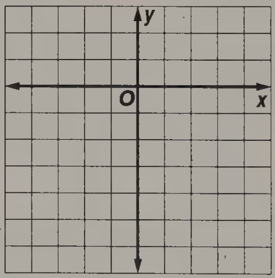


Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

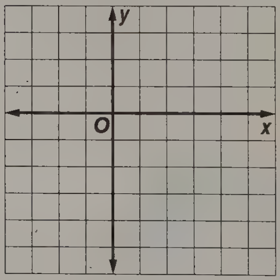
5. $y = 2x^2$



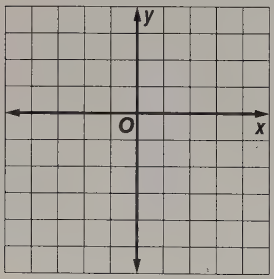
6. $y = x^2 - 2x - 5$



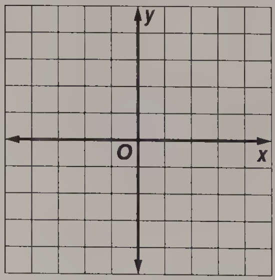
7. $y = -x^2 + 4x - 1$



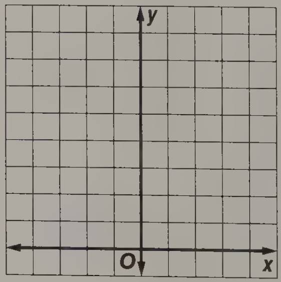
8. $y = -x^2 - 2x + 2$



9. $y = 2x^2 + 4x - 2$



10. $y = -2x^2 - 4x + 6$



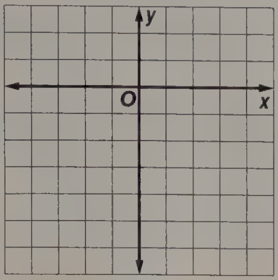
NAME ______________________ DATE ____________ PERIOD ____

9-1 Practice

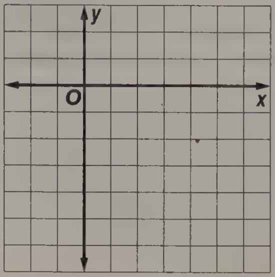
Graphing Quadratic Functions

Use a table of values to graph each function. Determine the domain and range.

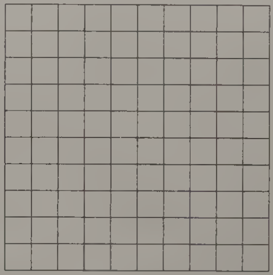
1. $y = -x^2 + 2$



2. $y = x^2 - 6x + 3$

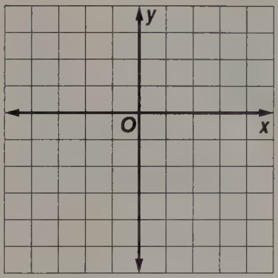


3. $y = -2x^2 - 8x - 5$

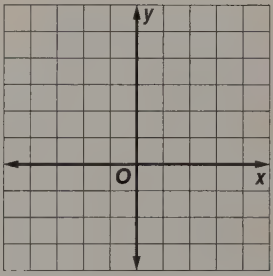


Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

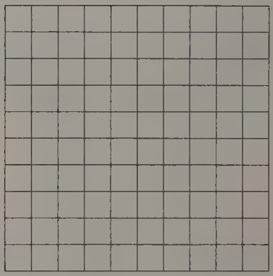
4. $y = -x^2 + 3$



5. $y = -2x^2 + 8x - 3$



6. $y = 2x^2 + 8x + 1$



PHYSICS For Exercises 7–9, use the following information.

Miranda throws a set of keys up to her brother, who is standing on a balcony 38 feet above the ground. The equation $h = -16t^2 + 40t + 5$ gives the height h of the keys after t seconds.

7. How long does it take the keys to reach their highest point?

8. How high do the keys reach?

9. Will her brother be able to catch the keys? Explain.

10. What is a reasonable domain and range for this problem?

BASEBALL For Exercises 11–13, use the following information.

A player hits a baseball into the outfield. The equation $h = -0.005x^2 + x + 3$ gives the path of the ball, where h is the height and x is the horizontal distance the ball travels.

11. What is the equation of the axis of symmetry?

12. What is the maximum height reached by the baseball?

13. An outfielder catches the ball three feet above the ground. How far has the ball traveled horizontally when the outfielder catches it?

9-1 Word Problem Practice

Graphing Quadratic Functions

1. **OLYMPICS** Olympics were held in 1896 and have been held every four years (except 1916, 1940, and 1944). The winning height y in men's pole vault at any number Olympiad x can be approximated by the equation $y = 0.37x^2 + 4.3x + 126$. Complete the table to estimate the pole vault heights in each of the Olympic Games. Round your answers to the nearest tenth.

Year	Olympiad (x)	Height (y inches)
1896	1	
1900	2	
1924	7	
1936	10	
1964	15	
2008	26	

Source: www.nsa.gov/teachers/hs/alg01.pdf

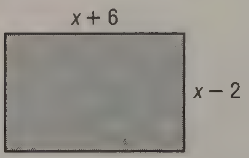
2. **PHYSICS** Mrs. Capwell's physics class investigates what happens when a ball is given an initial push, rolls up, and then back down an inclined plane. The class finds that $y = -x^2 + 6x$ accurately predicts the ball's position y after rolling x seconds. On the graph of the equation, what would be the y value when $x = 4$?

3. **ARCHITECTURE** A hotel's main entrance is in the shape of a parabolic arch. The equation $y = -x^2 + 10x$ models the arch height y for any distance x from one side of the arch. Use a graph to determine its maximum height.

4. **SOFTBALL** Olympic softball gold medalist Michele Smith pitches a curveball with a speed of 64 feet per second. If she throws the ball straight upward at this speed, the ball's height h (in feet) after t seconds is given by $h = -16t^2 + 64t$. Find the coordinates of the vertex of the graph of the ball's height and interpret its meaning.

GEOMETRY For Exercises 5–7, use the following information.

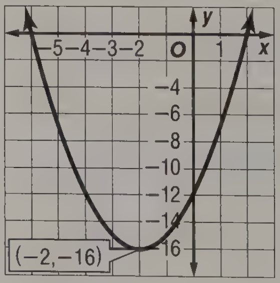
Teddy is building the rectangular deck shown below.



5. Write the equation representing the area of the deck.

6. What is the equation of the axis of symmetry?

7. Graph the equation and label its vertex.



NAME ______________________ DATE ____________ PERIOD _____

9-1 Enrichment

Lesson 9-1

Translating Quadratic Graphs

When a figure is moved to a new position without undergoing any rotation, then the figure is said to have been **translated** to that position.

The graph of a quadratic equation in the form $y = (x - b)^2 + c$ is a translation of the graph of $y = x^2$.

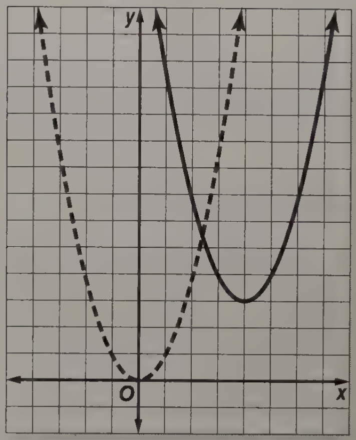
Start with $y = x^2$.

Slide to the right 4 units.

$y = (x - 4)^2$

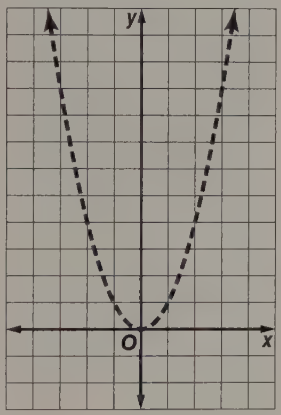
Then slide up 3 units.

$y = (x - 4)^2 + 3$

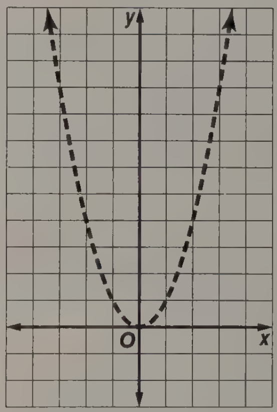


These equations have the form $y = x^2 + c$. Graph each equation.

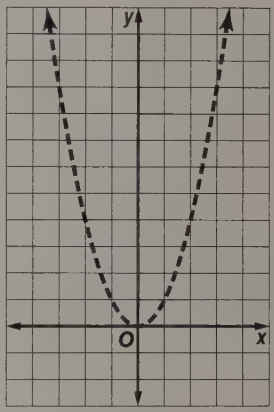
1. $y = x^2 + 1$



2. $y = x^2 + 2$

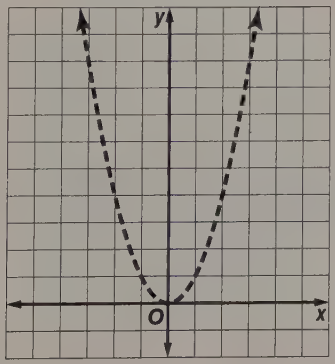


3. $y = x^2 - 2$

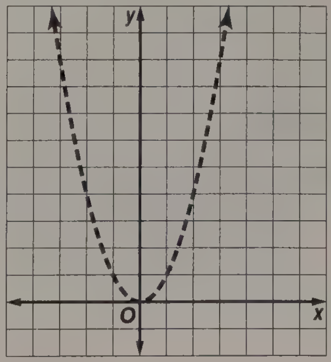


These equations have the form $y = (x - b)^2$. Graph each equation.

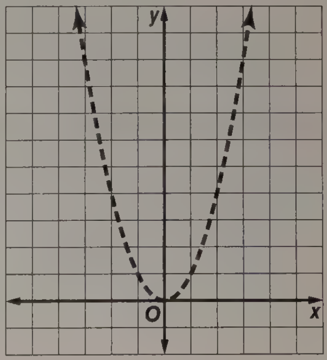
4. $y = (x - 1)^2$



5. $y = (x - 3)^2$



6. $y = (x + 2)^2$



NAME ______________________________ DATE ____________ PERIOD _____

9-2 Lesson Reading Guide

Solving Quadratic Equations by Graphing

Get Ready for the Lesson

Read the introduction to Lesson 9-2 in your textbook.

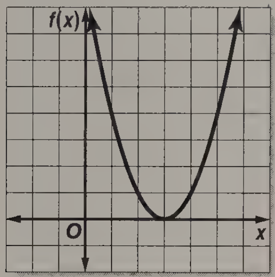
If one of the x-intercepts represents the location where the ball will hit the ground, what does the other x-intercept represent?

Read the Lesson

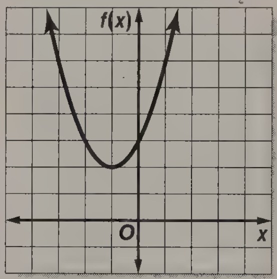
1. The x-intercepts of the graph of a quadratic function are the x-coordinates of the points where the graph of the function intersects the x-axis. At those points, the y-coordinates are equal to __________. This explains why the x-intercepts are called __________ of the quadratic function.

2. The graphs of three functions are shown below. Use the graphs to provide the requested information about the related quadratic equations.

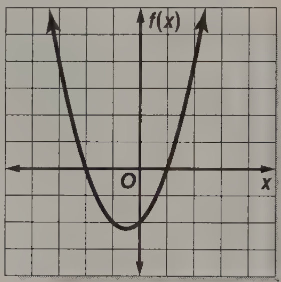
A. $f(x) = x^2 - 6x + 9$



B. $f(x) = x^2 + 2x + 3$



C. $f(x) = x^2 + x - 2$



a. For Graph A, the related quadratic equation is ______________________.

How many real solutions are there?

Name any solutions.

b. For Graph B, the related quadratic equation is ______________________.

How many real solutions are there?

Name any solutions.

c. For Graph C, the related quadratic equation is ______________________.

How many real solutions are there?

Name any solutions.

Remember What You Learned

3. Describe how you can remember that the word *zero* is used when you are talking about functions, but the word *root* is used when you are talking about equations.

NAME ______________________ DATE ____________ PERIOD _____

9-2 Study Guide and Intervention

Solving Quadratic Equations by Graphing

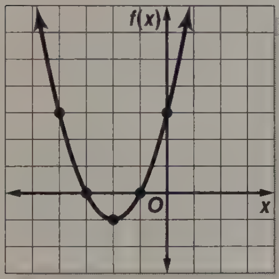
Solve by Graphing

Quadratic Equation	an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$

The solutions of a quadratic equation are called the **roots** of the equation. The roots of a quadratic equation can be found by graphing the related quadratic function $f(x) = ax^2 + bx + c$ and finding the x-intercepts or **zeros** of the function.

Example 1 **Solve $x^2 + 4x + 3 = 0$ by graphing.**

Graph the related function $f(x) = x^2 + 4x + 3$. The equation of the axis of symmetry is $x = -\frac{4}{2(1)}$ or -2. The vertex is at $(-2, -1)$. Graph the vertex and several other points on either side of the axis of symmetry.

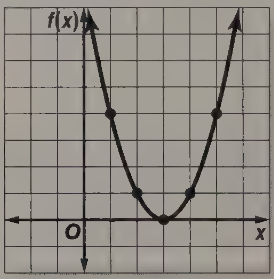


To solve $x^2 + 4x + 3 = 0$, you need to know where the value of $f(x) = 0$. This occurs at the x-intercepts, -3 and -1.

The solutions are -3 and -1.

Example 2 **Solve $x^2 - 6x + 9 = 0$ by graphing.**

Graph the related function $f(x) = x^2 - 6x + 9$. The equation of the axis of symmetry is $x = \frac{6}{2(1)}$ or 3. The vertex is at $(3, 0)$. Graph the vertex and several other points on either side of the axis of symmetry.

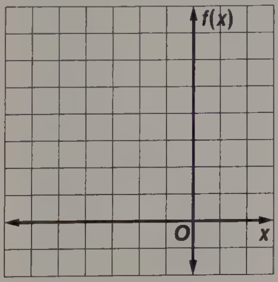


To solve $x^2 - 6x + 9 = 0$, you need to know where the value of $f(x) = 0$. The vertex of the parabola is the x-intercept. Thus, the only solution is 3.

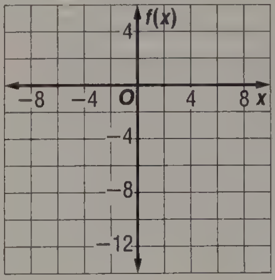
Exercises

Solve each equation by graphing.

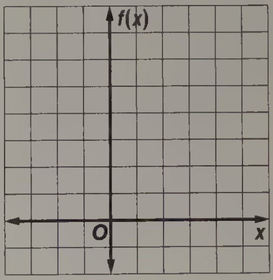
1. $x^2 + 7x + 12 = 0$



2. $x^2 - x - 12 = 0$



3. $x^2 - 4x + 5 = 0$



NAME ______________________________ DATE ____________ PERIOD _____

9-2 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Graphing

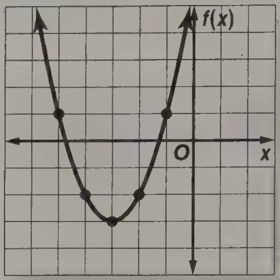
Estimate Solutions The roots of a quadratic equation may not be integers. If exact roots cannot be found, they can be estimated by finding the consecutive integers between which the roots lie.

Example **Solve $x^2 + 6x + 6 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.**

Graph the related function $f(x) = x^2 + 6x + 6$.

x	$f(x)$
−5	1
−4	−2
−3	−3
−2	−2
−1	1

Notice that the value of the function changes from negative to positive between the x-values of −5 and −4 and between −2 and −1.

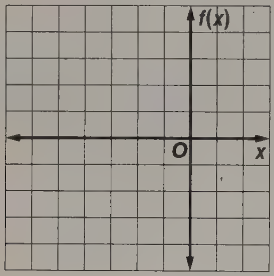


The x-intercepts of the graph are between −5 and −4 and between −2 and −1. So one root is between −5 and −4, and the other root is between −2 and −1.

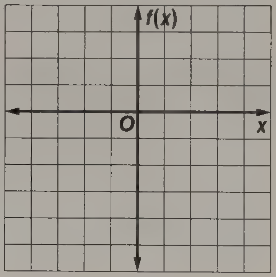
Exercises

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

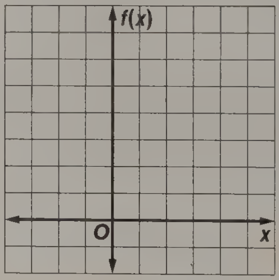
1. $x^2 + 7x + 9 = 0$



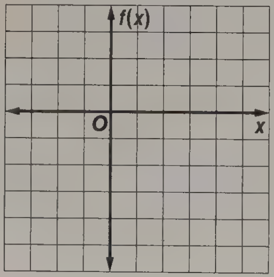
2. $x^2 - x - 4 = 0$



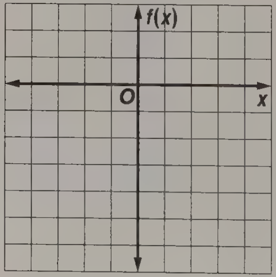
3. $x^2 - 4x + 6 = 0$



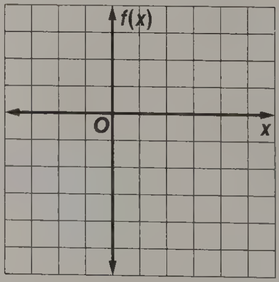
4. $x^2 - 4x - 1 = 0$



5. $4x^2 - 12x + 3 = 0$



6. $x^2 - 2x - 4 = 0$



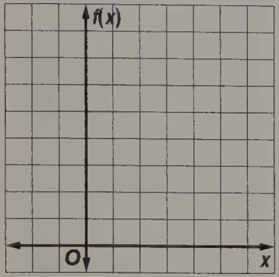
NAME ______________________________ DATE ____________ PERIOD _____

9-2 Skills Practice

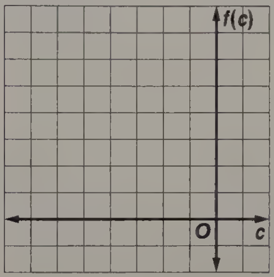
Solving Quadratic Equations by Graphing

Solve each equation by graphing.

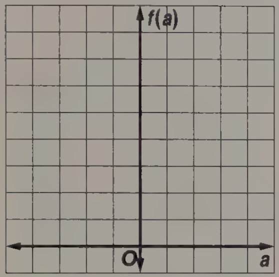
1. $x^2 - 2x + 3 = 0$



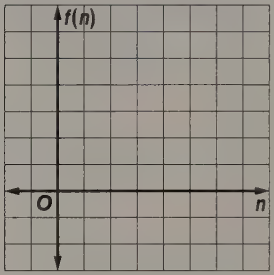
2. $c^2 + 6c + 8 = 0$



3. $a^2 - 2a = -1$

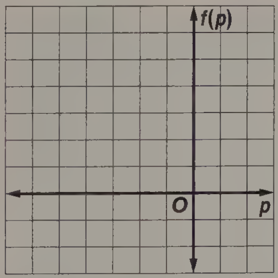


4. $n^2 - 7n = -10$

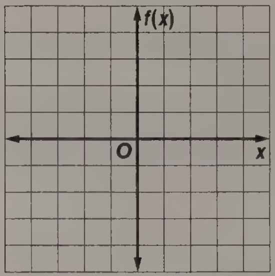


Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

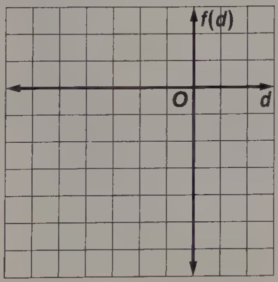
5. $p^2 + 4p + 2 = 0$



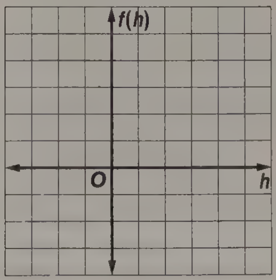
6. $x^2 + x - 3 = 0$



7. $d^2 + 6d = -3$



8. $h^2 + 1 = 4h$



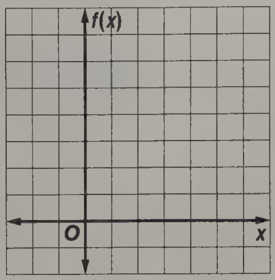
NAME ______________________ DATE ____________ PERIOD ______

9-2 Practice

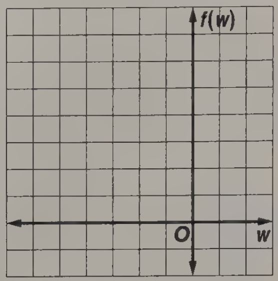
Solving Quadratic Equations by Graphing

Solve each equation by graphing.

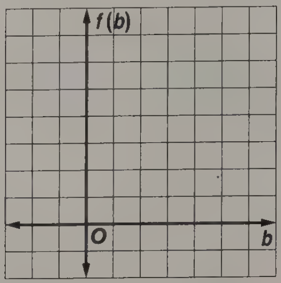
1. $x^2 - 5x + 6 = 0$



2. $w^2 + 6w + 9 = 0$

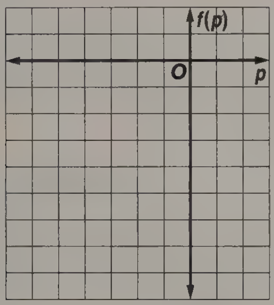


3. $b^2 - 3b + 4 = 0$

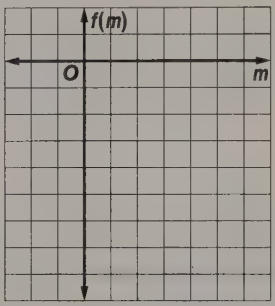


Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

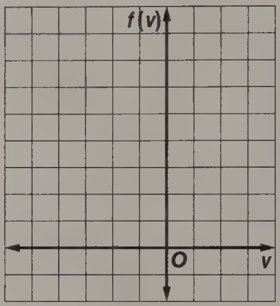
4. $p^2 + 4p = 3$



5. $2m^2 + 5 = 10m$



6. $2v^2 + 8v = -7$

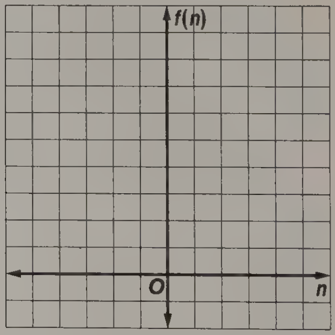


NUMBER THEORY For Exercises 7 and 8, use the following information.

Two numbers have a sum of 2 and a product of −8. The quadratic equation $-n^2 + 2n + 8 = 0$ can be used to determine the two numbers.

7. Graph the related function $f(n) = -n^2 + 2n + 8$ and determine its x-intercepts.

8. What are the two numbers?

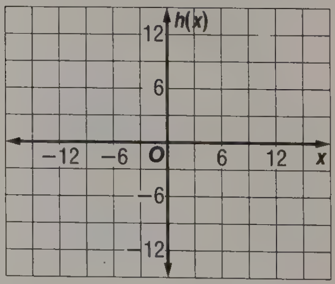


DESIGN For Exercises 9 and 10, use the following information.

A footbridge is suspended from a parabolic support. The function $h(x) = -\frac{1}{25}x^2 + 9$ represents the height in feet of the support above the walkway, where $x = 0$ represents the midpoint of the bridge.

9. Graph the function and determine its x-intercepts.

10. What is the length of the walkway between the two supports?

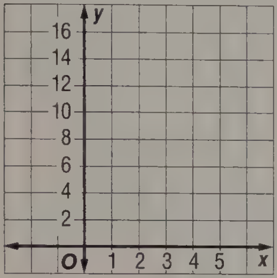


NAME ______________________ DATE ____________ PERIOD _____

9-2 Word Problem Practice

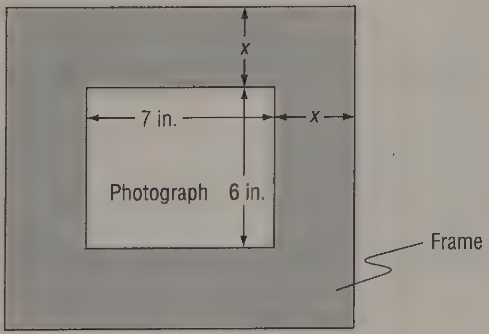
Solving Quadratic Equations by Graphing

1. **FARMING** In order for Ray to decide how much fertilizer to apply to his corn crop this year, he reviews records from previous years. He finds that his crop yield y depends on the amount of fertilizer he applies to his fields x according to the equation $y = -x^2 + 4x + 12$. Graph the function, and find the point at which Ray gets the highest yield possible.



2. **LIGHT** Ayzha and Jeremy hold a flashlight so that the light falls on a piece of graph paper in the shape of a parabola. Ayzha and Jeremy sketch the shape of the parabola and find that the equation $y = x^2 - 3x - 10$ matches the shape of the light beam. Determine the roots of the function.

3. **FRAMING** A rectangular photograph is 7 inches long and 6 inches wide. The photograph is framed using material x inches wide. If the area of the frame and photograph combined is 156 square inches, what is the width of the framing material?



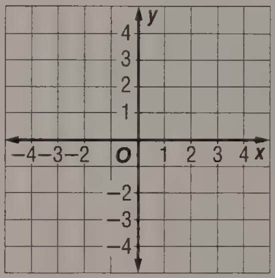
4. **WRAPPING PAPER** Can a rectangular piece of wrapping paper with an area of 81 square inches have a perimeter of 60 inches? (*Hint:* Let length = $30 - w$.) Explain.

ENGINEERING For Exercises 5–7, use the following information.

The shape of a satellite dish is often parabolic because of the reflective qualities of parabolas. Suppose a particular satellite dish is modeled by the following equation.

$$0.5x^2 = 2 + y$$

5. Approximate the solution by graphing.



6. On the coordinate plane above, translate the parabola so that there is a double root. Label this curve A.

7. Translate the parabola so that there are no roots. Label this curve B.

NAME ______________________ DATE ____________ PERIOD _____

9-2 Enrichment

Rational Exponents

You have developed the following properties of powers when a is a positive real number and m and n are integers.

$a^m \cdot a^n = a^{m+n}$ $\qquad (ab)^m = a^m b^m$ $\qquad a^0 = 1$

$(a^m)^n = a^{mn}$ $\qquad \frac{a^m}{a^n} = a^{m-n}$ $\qquad a^{-m} = \frac{1}{a^m}$

Exponents need not be restricted to integers. We can define rational exponents so that operations involving them will be governed by the properties for integer exponents.

$\left(a^{\frac{1}{2}}\right)^2 = a^{\frac{1}{2} \cdot 2} = a$ $\qquad \left(a^{\frac{1}{3}}\right)^3 = a^{\frac{1}{3} \cdot 3}$ $\qquad \left(a^{\frac{1}{n}}\right)^n = a^{\frac{1}{n} \cdot n} = a$

$a^{\frac{1}{2}}$ squared is a. $\qquad a^{\frac{1}{3}}$ cubed is a. $\qquad a^{\frac{1}{n}}$ to the n power is a.

$a^{\frac{1}{2}}$ is a square root of a. $\qquad a^{\frac{1}{3}}$ is a cube root of a. $\qquad a^{\frac{1}{n}}$ is an nth root of a.

$a^{\frac{1}{2}} = \sqrt{a}$ $\qquad a^{\frac{1}{3}} = \sqrt[3]{a}$ $\qquad a^{\frac{1}{n}} = \sqrt[n]{a}$

Now let us investigate the meaning of $a^{\frac{m}{n}}$.

$a^{\frac{m}{n}} = a^{m \cdot \frac{1}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$ $\qquad a^{\frac{m}{n}} = a^{\frac{1}{n} \cdot m} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$

Therefore, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m$.

Example 1 **Write $\sqrt[4]{a^3}$ in exponential form.**

$\sqrt[4]{a^3} = a^{\frac{3}{4}}$

Example 2 **Write $a^{\frac{2}{5}}$ in radical form.**

$a^{\frac{2}{5}} = \sqrt[5]{a^2}$

Example 3 **Find $\frac{a^{\frac{2}{3}}}{a^{\frac{1}{2}}}$.**

$\frac{a^{\frac{2}{3}}}{a^{\frac{1}{2}}} = a^{\frac{2}{3} - \frac{1}{2}} = a^{\frac{4}{6} - \frac{3}{6}} = a^{\frac{1}{6}}$ or $\sqrt[6]{a}$

Write each expression in radical form.

1. $b^{\frac{2}{3}}$

2. $3c^{\frac{1}{2}}$

3. $(3c)^{\frac{1}{2}}$

Write each expression in exponential form.

4. $\sqrt[3]{b^4}$

5. $\sqrt{4a^3}$

6. $2 \cdot \sqrt[3]{b^2}$

Perform the operation indicated. Answers should show positive exponents only.

7. $\left(a^3 b^{\frac{1}{4}}\right)^2$

8. $\frac{-8a^{\frac{3}{4}}}{2a^{\frac{1}{2}}}$

9. $\left(\frac{b^{\frac{1}{2}}}{b^{-\frac{2}{3}}}\right)^3$

10. $\sqrt{a^3} \cdot \sqrt{a}$

11. $\left(a^2 b^{-\frac{1}{3}}\right)^{-\frac{1}{2}}$

12. $-2a^{\frac{1}{3}}b^0\left(5a^{\frac{1}{2}}b^{-\frac{2}{3}}\right)$

NAME ______________________ DATE ____________ PERIOD ____

9-3 Lesson Reading Guide

Solving Quadratic Equations by Completing the Square

Get Ready for the Lesson

Read the introduction to Lesson 9-3 in your textbook.

To solve the problem, how many "units" would Al-Khwarizmi have added to each side of the equation?

Read the Lesson

1. Draw a line under each quadratic equation that you could solve by taking the square root of each side.

$x^2 + 6x + 9 = 100$ $\quad$ $x^2 - 14x + 40 = 25$ $\quad$ $x^2 - 16x + 64 = 26$

$x^2 - 20x + 80 = 16$ $\quad$ $x^2 + 10x + 36 = 49$ $\quad$ $x^2 - 12x + 36 = 6$

2. How can you tell whether it is possible to solve a quadratic equation by taking the square root of each side?

3. Explain how to find what number is needed for the ■ in order to make $x^2 - 20x +$ ■ a perfect square.

4. To solve $3x^2 - 6x = 54$ by completing the square, why does it help first to divide both sides by 3?

Remember What You Learned

5. The method of completing the square might be easier to remember if you can connect it to what you know about perfect square trinomials. How is completing the square related to the method you use to determine whether a trinomial is a perfect square trinomial?

NAME ______________________________ DATE ____________ PERIOD _____

9-3 Study Guide and Intervention

Solving Quadratic Equations by Completing the Square

Find the Square Root An equation such as $x^2 - 4x + 4 = 5$ can be solved by taking the square root of each side.

Example 1 **Solve $x^2 - 2x + 1 = 9$. Round to the nearest tenth if necessary.**

$$x^2 - 2x + 1 = 9$$
$$(x - 1)^2 = 9$$
$$\sqrt{(x - 1)^2} = \sqrt{9}$$
$$|x - 1| = \sqrt{9}$$
$$x - 1 = \pm 3$$
$$x - 1 + 1 = \pm 3 + 1$$
$$x = 1 \pm 3$$

$x = 1 + 3$ or $x = 1 - 3$

$= 4$ $= -2$

The solution set is $\{-2, 4\}$.

Example 2 **Solve $x^2 - 4x + 4 = 5$. Round to the nearest tenth if necessary.**

$$x^2 - 4x + 4 = 5$$
$$(x - 2)^2 = 5$$
$$\sqrt{(x - 2)^2} = \sqrt{5}$$
$$|x - 2| = \sqrt{5}$$
$$x - 2 = \pm\sqrt{5}$$
$$x - 2 + 2 = \pm\sqrt{5} + 2$$
$$x = 2 \pm \sqrt{5}$$

Use a calculator to evaluate each value of x.

$x = 2 + \sqrt{5}$ or $x = 2 - \sqrt{5}$

≈ 4.2 ≈ -0.2

The solution set is $\{-0.2, 4.2\}$.

Exercises

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $x^2 + 4x + 4 = 9$

2. $m^2 + 12m + 36 = 1$

3. $r^2 - 6r + 9 = 16$

4. $x^2 - 2x + 1 = 25$

5. $x^2 - 8x + 16 = 5$

6. $x^2 - 10x + 25 = 8$

7. $c^2 - 4c + 4 = 7$

8. $p^2 + 16p + 64 = 3$

9. $x^2 + 8x + 16 = 9$

10. $x^2 + 6x + 9 = 4$

11. $a^2 + 8a + 16 = 10$

12. $y^2 - 12y + 36 = 5$

13. $x^2 + 10x + 25 = 1$

14. $y^2 + 14y + 49 = 6$

15. $m^2 - 8m + 16 = 2$

16. $x^2 + 12x + 36 = 10$

17. $a^2 - 14a + 49 = 3$

18. $y^2 + 8y + 16 = 7$

9-3 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Completing the Square

Complete the Square Since few quadratic expressions are perfect square trinomials, the method of **completing the square** can be used to solve some quadratic equations. Use the following steps to complete the square for a quadratic expression of the form $ax^2 + bx$.

Step 1	Find $\frac{b}{2}$.
Step 2	Find $\left(\frac{b}{2}\right)^2$.
Step 3	Add $\left(\frac{b}{2}\right)^2$ to $ax^2 + bx$.

Example **Solve $x^2 + 6x + 3 = 10$ by completing the square.**

$x^2 + 6x + 3 = 10$	Original equation
$x^2 + 6x + 3 - 3 = 10 - 3$	Subtract 3 from each side.
$x^2 + 6x = 7$	Simplify.
$x^2 + 6x + 9 = 7 + 9$	Since $\left(\frac{6}{2}\right)^2 = 9$, add 9 to each side.
$(x + 3)^2 = 16$	Factor $x^2 + 6x + 9$.
$x + 3 = \pm 4$	Take the square root of each side.
$x = -3 \pm 4$	Simplify.

$x = -3 + 4$ or $x = -3 - 4$

$= 1$ $\quad$ $= -7$

The solution set is $\{-7, 1\}$.

Exercises

Solve each equation by completing the square. Round to the nearest tenth if necessary.

1. $t^2 - 4t + 3 = 0$

2. $y^2 + 10y = -9$

3. $y^2 - 8y - 9 = 0$

4. $x^2 - 6x = 16$

5. $p^2 - 4p - 5 = 0$

6. $x^2 - 12x = 9$

7. $c^2 + 8c = 20$

8. $p^2 = 2p + 1$

9. $x^2 + 20x + 11 = -8$

10. $x^2 - 1 = 5x$

11. $a^2 = 22a + 23$

12. $m^2 - 8m = -7$

13. $x^2 + 10x = 24$

14. $a^2 - 18a = 19$

15. $b^2 + 16b = -16$

16. $4x^2 = 24 + 4x$

17. $2m^2 + 4m + 2 = 8$

18. $4k^2 = 40k + 44$

Lesson 9-3

NAME ______________________ DATE ____________ PERIOD ____

9-3 Skills Practice

Solving Quadratic Equations by Completing the Square

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $c^2 - 12c + 36 = 4$

2. $w^2 - 10w + 25 = 16$

3. $b^2 + 16b + 64 = 9$

4. $y^2 + 2y + 1 = 3$

5. $r^2 + 4r + 4 = 7$

6. $a^2 - 8a + 16 = 12$

Find the value of *c* that makes each trinomial a perfect square.

7. $g^2 + 6g + c$

8. $y^2 + 4y + c$

9. $a^2 - 14a + c$

10. $n^2 - 2n + c$

11. $s^2 - 18s + c$

12. $p^2 + 20p + c$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

13. $x^2 + 4x - 12 = 0$

14. $v^2 - 8v + 15 = 0$

15. $q^2 + 6q = 7$

16. $r^2 - 2r = 15$

17. $m^2 - 14m + 30 = 6$

18. $b^2 + 12b + 21 = 10$

19. $z^2 - 4z + 1 = 0$

20. $y^2 - 6y + 4 = 0$

21. $r^2 - 8r + 10 = 0$

22. $p^2 - 2p = 5$

23. $2a^2 + 20a = -2$

24. $0.5g^2 + 8g = -7$

NAME ______________________ DATE ____________ PERIOD _____

9-3 Practice

Solving Quadratic Equations by Completing the Square

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $b^2 - 14b + 49 = 64$

2. $s^2 + 16s + 64 = 100$

3. $h^2 - 8h + 16 = 15$

4. $a^2 + 6a + 9 = 27$

5. $p^2 - 20p + 100 = 28$

6. $u^2 + 10u + 25 = 90$

Find the value of c that makes each trinomial a perfect square.

7. $t^2 - 24t + c$

8. $b^2 + 28b + c$

9. $y^2 + 40y + c$

10. $m^2 + 3m + c$

11. $g^2 - 9g + c$

12. $v^2 - v + c$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

13. $w^2 - 14w + 24 = 0$

14. $p^2 + 12p = 13$

15. $s^2 - 30s + 56 = -25$

16. $v^2 + 8v + 9 = 0$

17. $t^2 - 10t + 6 = -7$

18. $n^2 + 18n + 50 = 9$

19. $3u^2 + 15u - 3 = 0$

20. $4c^2 - 72 = 24c$

21. $0.9a^2 + 5.4a - 4 = 0$

22. $0.4h^2 + 0.8h = 0.2$

23. $\frac{1}{2}x^2 - \frac{1}{2}x - 10 = 0$

24. $\frac{1}{4}x^2 + \frac{3}{2}x - 2 = 0$

BUSINESS For Exercises 25 and 26, use the following information.

Jaime owns a business making decorative boxes to store jewelry, mementos, and other valuables. The function $y = x^2 + 50x + 1800$ models the profit y that Jaime has made in month x for the first two years of his business.

25. Write an equation representing the month in which Jaime's profit is $2400.

26. Use completing the square to find out in which month Jaime's profit is $2400.

27. PHYSICS From a height of 256 feet above a lake on a cliff, Mikaela throws a rock out over the lake. The height H of the rock t seconds after Mikaela throws it is represented by the equation $H = -16t^2 + 32t + 256$. To the nearest tenth of a second, how long does it take the rock to reach the lake below? (*Hint:* Replace H with 0.)

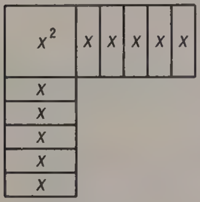
Lesson 9-3

NAME ______________________________ DATE ____________ PERIOD ______

9-3 Word Problem Practice

Solving Quadratic Equations by Completing the Square

1. **INTERIOR DESIGN** Modular carpeting is installed in small pieces rather than as a large roll so that only a few pieces need to be replaced if a small area is damaged. Suppose the room shown in the diagram below is being fitted with modular carpeting. Complete the square to determine the number of 1 ft by 1 ft squares of carpeting needed to finish the room. Fill in the missing terms in the corresponding equation below.



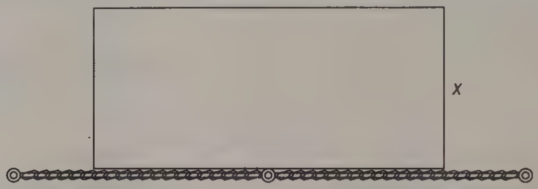
$x^2 + 10x +$ _____ $= (x +$ _____$)^2$

2. **FALLING OBJECTS** Keisha throws a rock down an old well. The distance d (in feet) the rock falls after t seconds can be represented by the equation $d = 16t^2 + 64t$. If the water in the well is 80 feet below ground, how many seconds will it take for the rock to hit the water?

3. **MARS** On Mars, the gravity acting on an object is less than that on Earth. On Earth, a golf ball hit with an initial upward velocity of 26 meters per second will reach a maximum height of about 34.5 meters. The height h of an object on Mars that leaves the ground with an initial velocity of 26 meters per second is given by the equation $h = -1.9t^2 + 26t$. Find the maximum height if the same golf ball is hit on Mars. Round your answer to the nearest tenth.

4. **FROGS** A frog sitting on a stump 3 feet high hops off and lands on the ground. During its leap, its height h (in feet) is given by $h = -0.5d^2 + 2d + 3$, where d is the distance from the base of the stump. How far is the frog from the base of the stump when it landed on the ground?

GARDENING For exercises 5–7, use the following information.
Peg is planning a rectangular vegetable garden using 250 feet of fencing material. She only needs to fence three sides of the garden since one side borders an existing fence.



5. Let x = the width of the rectangle. Write an expression to represent the area of the garden if she uses all the fencing material.

6. Find the vertex of the equation and identify it as a maximum or a minimum.

7. Interpret the vertex of the equation in terms of the situation.

NAME ______________________ DATE ____________ PERIOD _____

9-3 Enrichment

Parabolas Through Three Given Points

If you know two points on a straight line, you can find the equation of the line. To find the equation of a parabola, you need three points on the curve.

Here is how to approximate an equation of the parabola through the points (0, −2), (3, 0), and (5, 2).

Use the general equation $y = ax^2 + bx + c$. By substituting the given values for x and y, you get three equations.

$$\begin{array}{ll} (0, -2): & -2 = c \\ (3, 0): & 0 = 9a + 3b + c \\ (5, 2): & 2 = 25a + 5b + c \end{array}$$

First, substitute −2 for c in the second and third equations. Then solve those two equations as you would any system of two equations. Multiply the second equation by 5 and the third equation by −3.

$$\begin{array}{lcrcl} 0 = 9a + 3b - 2 & \xrightarrow{\text{Multiply by 5.}} & 0 & = & 45a + 15b - 10 \\ 0 = 25a + 5b - 2 & \xrightarrow{\text{Multiply by } -3.} & -6 & = & -75a - 15b + 6 \\ \hline & & -6 & = & -30a \qquad\quad - 4 \\ & & a & = & \frac{1}{15} \end{array}$$

To find b, substitute $\frac{1}{15}$ for a in either the second or third equation.

$$0 = 9\left(\frac{1}{15}\right) + 3b - 2$$

$$b = \frac{7}{15}$$

The equation of a parabola through the three points is

$$y = \frac{1}{15}x^2 + \frac{7}{15}x - 2.$$

Find the equation of a parabola through each set of three points.

1. (1, 5), (0, 6), (2, 3)

2. (−5, 0), (0, 0), (8, 100)

3. (4, −4), (0, 1), (3, −2)

4. (1, 3), (6, 0), (0, 0)

5. (2, 2), (5, −3), (0, −1)

6. (0, 4), (4, 0), (−4, 4)

NAME ______________________________ DATE ______________ PERIOD ______

9-4 Lesson Reading Guide

Solving Quadratic Equations by Using the Quadratic Formula

Get Ready for the Lesson

Read the introduction to Lesson 9-4 in your textbook.

Your teacher asks you to predict when 17% of the population will consist of people born outside the United States. What equation should you use to make the prediction?

Read the Lesson

1. Suppose you want to solve $12x^2 + 7x = 15$ using the Quadratic Formula.

a. What should you do first?

b. What are the values you need to substitute for a, b, and c in the Quadratic Formula?

c. Apply the Quadratic Formula using the above values, but do not solve the equation.

2. a. You can use the discriminant to determine the number of real roots for a quadratic equation. What is the discriminant?

b. Complete the statements below so that each statement is true.

When the value of the discriminant is ______________, there is one real root.

When the value of the discriminant is ______________, there are two real roots.

When the value of the discriminant is ______________, there are no real roots.

Remember What You Learned

3. To help remember the methods for solving a quadratic equation, explain how you would choose the best method for solving a quadratic equation $ax^2 + bx + c = 0$.

NAME ______________________ DATE ____________ PERIOD _____

9-4 Study Guide and Intervention

Solving Quadratic Equations by Using the Quadratic Formula

Quadratic Formula To solve the standard form of the quadratic equation, $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

Quadratic Formula	the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ that gives the solutions of $ax^2 + bx + c = 0$, where $a \neq 0$

Example 1 **Solve $x^2 + 2x = 3$ by using the Quadratic Formula.**

Rewrite the equation in standard form.

$x^2 + 2x = 3$ Original equation

$x^2 + 2x - 3 = 3 - 3$ Subtract 3 from each side.

$x^2 + 2x - 3 = 0$ Simplify.

Now let $a = 1$, $b = 2$, and $c = -3$ in the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{16}}{2}$$

$$x = \frac{-2 + 4}{2} \quad \text{or} \quad x = \frac{-2 - 4}{2}$$

$$= 1 \qquad\qquad = -3$$

The solution set is $\{-3, 1\}$.

Example 2 **Solve $x^2 - 6x - 2 = 0$ by using the Quadratic Formula. Round to the nearest tenth if necessary.**

For this equation $a = 1$, $b = -6$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{44}}{2}$$

$$x = \frac{6 + \sqrt{44}}{2} \quad \text{or} \quad x = \frac{6 - \sqrt{44}}{2}$$

$$\approx 6.3 \qquad\qquad \approx -0.3$$

The solution set is $\{-0.3, 6.3\}$.

Exercises

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 3x + 2 = 0$

2. $m^2 - 8m = -16$

3. $16r^2 - 8r = -1$

4. $x^2 + 5x = 6$

5. $3x^2 + 2x = 8$

6. $8x^2 - 8x - 5 = 0$

7. $-4c^2 + 19c = 21$

8. $2p^2 + 6p = 5$

9. $48x^2 + 22x - 15 = 0$

10. $8x^2 - 4x = 24$

11. $2p^2 + 5p = 8$

12. $8y^2 + 9y - 4 = 0$

13. $2x^2 + 9x + 4 = 0$

14. $8y^2 + 17y + 2 = 0$

15. $3z^2 + 5z - 2 = 0$

16. $-2x^2 + 8x + 4 = 0$

17. $a^2 + 3a = 2$

18. $2y^2 - 6y + 4 = 0$

NAME ______________________ DATE ____________ PERIOD ______

9-4 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Using the Quadratic Formula

The Discriminant In the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The discriminant can be used to determine the number of real roots for a quadratic equation.

Case 1: $b^2 - 4ac < 0$	Case 2: $b^2 - 4ac = 0$	Case 3: $b^2 - 4ac > 0$
no real roots	one real root	two real roots

Example **State the value of the discriminant for each equation. Then determine the number of real roots.**

a. $\mathbf{12x^2 + 5x = 4}$

Write the equation in standard form.

$12x^2 + 5x = 4$ Original equation

$12x^2 + 5x - 4 = 4 - 4$ Subtract 4 from each side.

$12x^2 + 5x - 4 = 0$ Simplify.

Now find the discriminant.

$b^2 - 4ac = (5)^2 - 4(12)(-4)$

$= 217$

Since the discriminant is positive, the equation has two real roots.

b. $\mathbf{2x^2 + 3x = -4}$

$2x^2 + 3x = -4$ Original equation

$2x^2 + 3x + 4 = -4 + 4$ Add 4 to each side.

$2x^2 + 3x + 4 = 0$ Simplify.

$b^2 - 4ac = (3)^2 - 4(2)(4)$

$= -23$

Since the discriminant is negative, the equation has no real roots.

Exercises

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

1. $3x^2 + 2x - 3 = 0$

2. $3n^2 - 7n - 8 = 0$

3. $2d^2 - 10d - 9 = 0$

4. $4x^2 = x + 4$

5. $3x^2 - 13x = 10$

6. $6x^2 - 10x + 10 = 0$

7. $2k^2 - 20 = -k$

8. $6p^2 = -11p - 40$

9. $9 - 18x + 9x^2 = 0$

10. $12x^2 + 9 = -6x$

11. $9a^2 = 81$

12. $16y^2 + 16y + 4 = 0$

13. $8x^2 + 9x = 2$

14. $4a^2 - 4a + 4 = 3$

15. $3b^2 - 18b = -14$

NAME ______________________________ DATE ____________ PERIOD _____

9-4 Skills Practice

Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $u^2 - 49 = 0$

2. $n^2 - n - 20 = 0$

3. $s^2 - 5s - 36 = 0$

4. $b^2 + 11b + 30 = 0$

5. $c^2 - 7c = -3$

6. $p^2 + 4p = -1$

7. $a^2 - 9a + 22 = 0$

8. $x^2 + 6x + 3 = 0$

9. $2x^2 + 5x - 7 = 0$

10. $2h^2 - 3h = -1$

11. $2p^2 + 5p + 4 = 0$

12. $2g^2 + 7g = 9$

13. $3t^2 + 2t - 3 = 0$

14. $3x^2 - 7x - 6 = 0$

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

15. $q^2 + 4q + 3 = 0$

16. $m^2 + 2m + 1 = 0$

17. $a^2 - 4a + 10 = 0$

18. $w^2 - 6w + 7 = 0$

19. $z^2 - 2z - 7 = 0$

20. $y^2 - 10y + 25 = 0$

21. $2d^2 + 5d - 8 = 0$

22. $2s^2 + 6s + 12 = 0$

23. $2u^2 - 4u + 10 = 0$

24. $3h^2 + 7h + 3 = 0$

Lesson 9-4

9-4 Practice

Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $g^2 + 2g - 3 = 0$

2. $a^2 + 8a + 7 = 0$

3. $v^2 - 4v + 6 = 0$

4. $d^2 - 6d + 7 = 0$

5. $2z^2 + 9z - 5 = 0$

6. $2r^2 + 12r + 10 = 0$

7. $2b^2 - 9b = -12$

8. $2h^2 - 5h = 12$

9. $3p^2 + p = 4$

10. $3m^2 - 1 = -8m$

11. $4y^2 + 7y = 15$

12. $1.6n^2 + 2n + 2.5 = 0$

13. $4.5k^2 + 4k - 1.5 = 0$

14. $\frac{1}{2}c^2 + 2c + \frac{3}{2} = 0$

15. $3w^2 - \frac{3}{4}w = \frac{1}{2}$

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

16. $a^2 + 8a + 16 = 0$

17. $c^2 + 3c + 12 = 0$

18. $2w^2 + 12w = -7$

19. $2u^2 + 15u = -30$

20. $4n^2 + 9 = 12n$

21. $3g^2 - 2g = 3.5$

22. $2.5k^2 + 3k - 0.5 = 0$

23. $\frac{3}{4}d^2 - 3d = -4$

24. $\frac{1}{4}s^2 = -s - 1$

CONSTRUCTION For Exercises 25 and 26, use the following information.

A roofer tosses a piece of roofing tile from a roof onto the ground 30 feet below. He tosses the tile with an initial downward velocity of 10 feet per second.

25. Write an equation to find how long it takes the tile to hit the ground. Use the model for vertical motion, $H = -16t^2 + vt + h$, where H is the height of an object after t seconds, v is the initial velocity, and h is the initial height. (*Hint:* Since the object is thrown down, the initial velocity is negative.)

26. How long does it take the tile to hit the ground?

27. PHYSICS Lupe tosses a ball up to Quyen, waiting at a third-story window, with an initial velocity of 30 feet per second. She releases the ball from a height of 6 feet. The equation $h = -16t^2 + 30t + 6$ represents the height h of the ball after t seconds. If the ball must reach a height of 25 feet for Quyen to catch it, does the ball reach Quyen? Explain. (*Hint:* Substitute 25 for h and use the discriminant.)

NAME ____________________ DATE ____________ PERIOD ______

9-4 Word Problem Practice

Solving Quadratic Equations by Using the Quadratic Formula

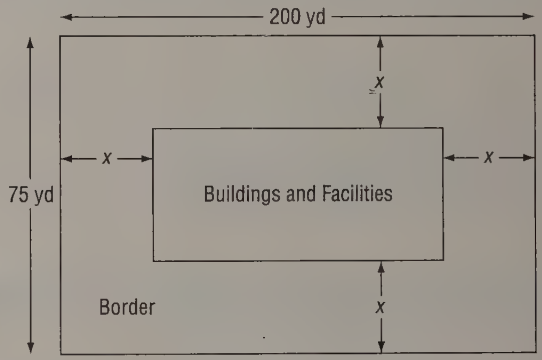
1. **BUSINESS** Tanya runs a catering business. Based on her records, her weekly profit can be approximated by the function $f(x) = x^2 + 2x - 37$, where x is the number of meals she caters. If $f(x)$ is negative, it means that the business has lost money. What is the least number of meals that Tanya needs to cater in order to have a profit?

2. **AERONAUTICS** At liftoff, the space shuttle *Discovery* has a constant acceleration of 16.4 feet per second squared and an initial velocity of 1341 feet per second due to the rotation of Earth. The distance *Discovery* has traveled t seconds after liftoff is given by the equation $d(t) = 1341t + 8.2t^2$. How long after liftoff has *Discovery* traveled 40,000 feet? Round your answer to the nearest tenth.

3. **ARCHITECTURE** The Golden Ratio appears in the design of the Greek Parthenon because the width and height of the façade are related by the equation $\frac{W + H}{W} = \frac{W}{H}$. If the height of a model of the Parthenon is 16 inches, what is its width? Round your answer to the nearest tenth.

4. **CRAFTS** Madelyn cut a 60-inch pipe cleaner into two unequal pieces, and then she used each piece to make a square. The sum of the areas of the squares was 117 square inches. Let x = the length of one piece. Write and solve an equation to represent the situation and find the lengths of the two original pieces.

SITE DESIGN For Exercises 5–7, refer to the following information. Round your answers to the nearest tenth.
The town of Smallport plans to build a new water treatment plant on a rectangular piece of land 75 yards wide and 200 yards long. The buildings and facilities need to cover an area of 10,000 square yards. The town's zoning board wants the site designer to allow as much room as possible between each edge of the site and the buildings and facilities. Let x = the width of the border.



5. Use an equation similar to area = length × width to represent the situation.

6. Write the equation in standard quadratic

7. What should be the width of the border?

Lesson 9-4

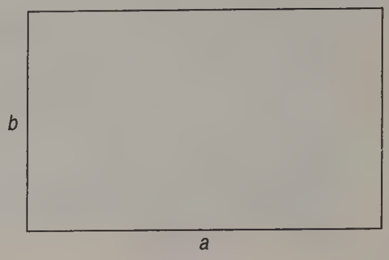
NAME ______________________ DATE ____________ PERIOD _____

9-4 Enrichment

Golden Rectangles

A **golden rectangle** has the property that its sides satisfy the following proportion.

$$\frac{a+b}{a} = \frac{a}{b}$$



Two quadratic equations can be written from the proportion. These are sometimes called **golden quadratic** equations.

1. In the proportion, let $a = 1$. Use cross-multiplication to write a quadratic equation.

2. Solve the equation in Exercise 1 for b.

3. In the proportion, let $b = 1$. Write a quadratic equation in a.

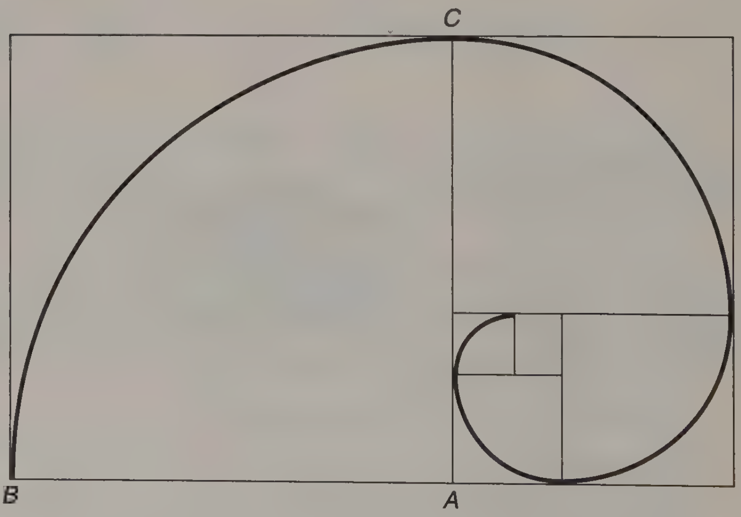
4. Solve the equation in Exercise 3 for a.

5. Explain why $\frac{1}{2}(\sqrt{5} + 1)$ and $\frac{1}{2}(\sqrt{5} - 1)$ are called golden ratios.

Another property of golden rectangles is that a square drawn inside a golden rectangle creates another, smaller golden rectangle.

In the design at the right, opposite vertices of each square have been connected with quarters of circles.

For example, the arc from point B to point C is created by putting the point of a compass at point A. The radius of the arc is the length BA.



6. On a separate sheet of paper, draw a larger version of the design. Start with a golden rectangle with a long side of 10 inches.

NAME ______________________ DATE ____________ PERIOD _____

9-5 Lesson Reading Guide

Exponential Functions

Get Ready for the Lesson

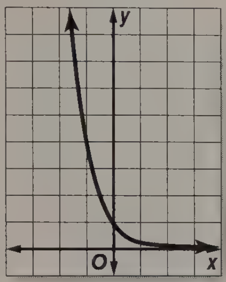
Read the introduction to Lesson 9-5 in your textbook.

If Mr. Warther had carved a ninth layer of pliers, how many pliers would he have carved?

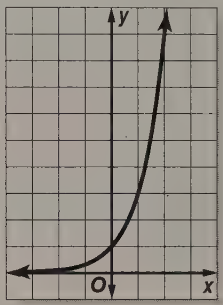
Read the Lesson

1. The graphs of two exponential functions of the form $y = a^x$ are shown below.

A.



B.



a. In Graph A, the value of a is greater than ________ and less than ________. The y values decrease as the x values ______________.

b. In Graph B, the value of a is greater than ________. The y values ______________ as the x values increase.

2. a. When you look for a pattern of exponential behavior in a set of data, what is the pattern you are looking for?

b. If a set of data has a negative common factor, does it display exponential behavior?

Remember What You Learned

3. What comparisons can you make between the quadratic function $y = x^2$ and the exponential function $y = 2^x$ to help remember the differences between quadratic and exponential functions?

NAME ______________________ DATE ____________ PERIOD _____

9-5 Study Guide and Intervention

Exponential Functions

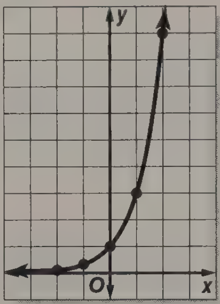
Graph Exponential Functions

Exponential Function	a function defined by an equation of the form $y = a^x$, where $a > 0$ and $a \neq 1$

You can use values of x to find ordered pairs that satisfy an exponential function. Then you can use the ordered pairs to graph the function.

Example 1 **Graph $y = 3^x$. State the y-intercept.**

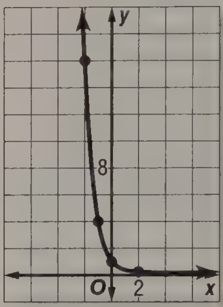
x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9



The y-intercept is 1.

Example 2 **Graph $y = \left(\frac{1}{4}\right)^x$. Use the graph to determine the approximate value of $\left(\frac{1}{4}\right)^{-0.5}$.**

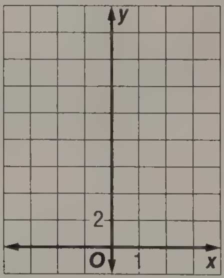
x	y
-2	16
-1	4
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$



The value of $\left(\frac{1}{4}\right)^{-0.5}$ is about 2.

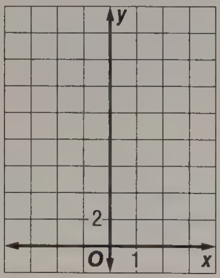
Exercises

1. Graph $y = 0.3^x$. State the y-intercept. Then use the graph to determine the approximate value of $0.3^{-1.5}$. Use a calculator to confirm the value.

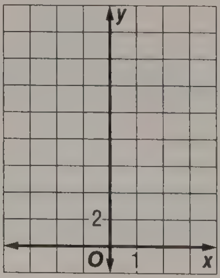


Graph each function. State the y-intercept.

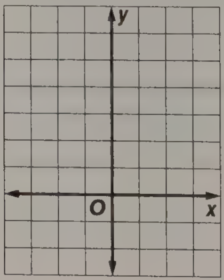
2. $y = 3^x + 1$



3. $y = \left(\frac{1}{3}\right)^x + 1$



4. $y = \left(\frac{1}{2}\right)^x - 2$



9-5 Study Guide and Intervention *(continued)*

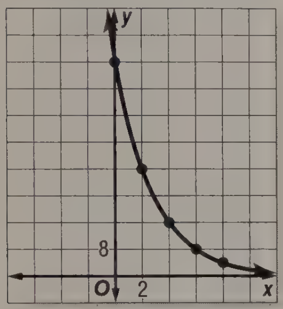
Exponential Functions

Identify Exponential Behavior It is sometimes useful to know if a set of data is exponential. One way to tell is to observe the shape of the graph. Another way is to observe the pattern in the set of data.

Example **Determine whether the set of data displays exponential behavior.**

x	0	2	4	6	8	10
y	64	32	16	8	4	2

Method 1: Graph the Data



The graph shows rapidly decreasing values of y as x increases. This is characteristic of exponential behavior.

Method 2: Look for a Pattern

The domain values increase by regular intervals of 2, while the range values have a common factor of $\frac{1}{2}$. Since the domain values increase by regular intervals and the range values have a common factor, the data are probably exponential.

Exercises

Determine whether the data in each table display exponential behavior. Explain why or why not.

1.

x	0	1	2	3
y	5	10	15	20

2.

x	0	1	2	3
y	3	9	27	81

3.

x	−1	1	3	5
y	32	16	8	4

4.

x	−1	0	1	2	3
y	3	3	3	3	3

5.

x	−5	0	5	10
y	1	0.5	0.25	0.125

6.

x	0	1	2	3	4
y	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$

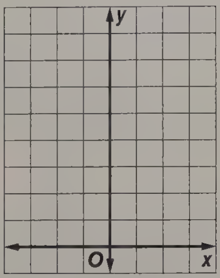
NAME ______________________________ DATE ____________ PERIOD _____

9-5 Skills Practice

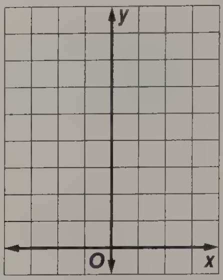
Exponential Functions

Graph each function. State the y-intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

1. $y = 2^x;\ 2^{2.3}$

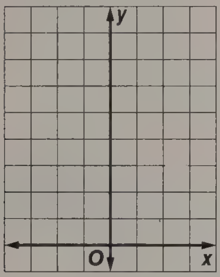


2. $y = \left(\frac{1}{3}\right)^x;\ \left(\frac{1}{3}\right)^{-1.6}$

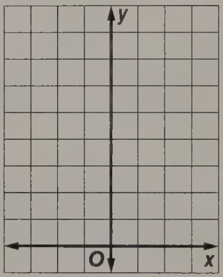


Graph each function. State the y-intercept.

3. $y = 3(2^x)$



4. $y = 3^x + 2$



Determine whether the data in each table display exponential behavior. Explain why or why not.

5.

x	−3	−2	−1	0
y	9	12	15	18

6.

x	0	5	10	15
y	20	10	5	2.5

7.

x	4	8	12	16
y	20	40	80	160

8.

x	50	30	10	−10
y	90	70	50	30

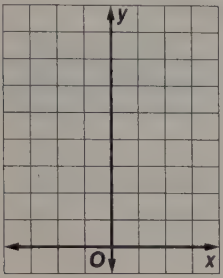
NAME ______________________________ DATE ____________ PERIOD _____

9-5 Practice

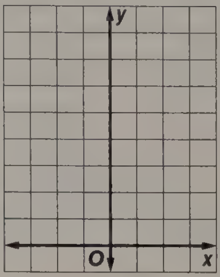
Exponential Functions

Graph each function. State the y-intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

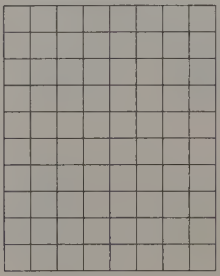
1. $y = \left(\frac{1}{10}\right)^x$; $\left(\frac{1}{10}\right)^{-0.5}$



2. $y = 3^x$; $3^{1.9}$

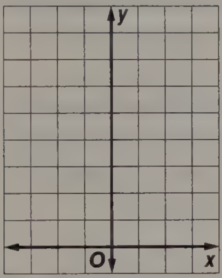


3. $y = \left(\frac{1}{4}\right)^x$; $\left(\frac{1}{4}\right)^{-1.4}$

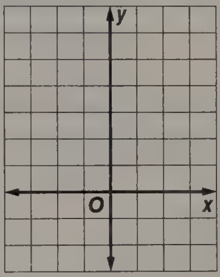


Graph each function. State the y-intercept.

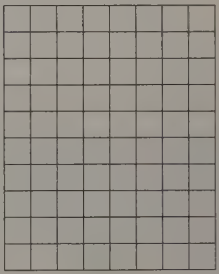
4. $y = 4(2^x) + 1$



5. $y = 2(2^x - 1)$



6. $y = 0.5(3^x - 3)$



Determine whether the data in each table display exponential behavior. Explain why or why not.

7.

x	2	5	8	11
y	480	120	30	7.5

8.

x	21	18	15	12
y	30	23	16	9

9. LEARNING Ms. Klemperer told her English class that each week students tend to forget one sixth of the vocabulary words they learned the previous week. Suppose a student learns 60 words. The number of words remembered can be described by the function $W(x) = 60\left(\frac{5}{6}\right)^x$, where x is the number of weeks that pass. How many words will the student remember after 3 weeks?

10. BIOLOGY Suppose a certain cell reproduces itself in four hours. If a lab researcher begins with 50 cells, how many cells will there be after one day, two days, and three days? (*Hint:* Use the exponential function $y = 50(2^x)$.)

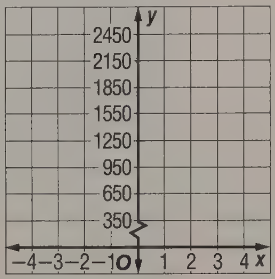
9-5 Word Problem Practice

Exponential Functions

1. **WASTE** Suppose the waste generated by nonrecycled paper and cardboard products is approximated by the following function.

$$y = 1000(2)^{0.3x}$$

Sketch the exponential function on the coordinate grid below.



2. **MONEY** Tatyana's grandfather gave her one penny on the day she was born. He plans to double the amount he gives her every day. Estimate how much she will receive from him on the 12th day of her life.

3. **PICTURE FRAMES** Since a picture frame includes a border, the picture must be smaller in area than the entire frame. The table shows the relationship between picture area and frame length for a particular line of frames. Is this an exponential relationship? Explain.

Side Length (in)	Picture Area (in^2)
5	6
6	12
7	20
8	30
9	42

4. **DEPRECIATION** The value of Royce Company's computer equipment is decreasing in value according to the following function.

$$y = 4000(0.87)^x$$

In the equation, x is the number of years that have elapsed since the equipment was purchased and y is in dollars. What was the value 5 years after it was purchased? Round your answer to the nearest dollar.

METEOROLOGY The atmospheric pressure (in millibars) at a given altitude x, in meters, can be approximated by the following function. The function is valid for values of x between 0 and 10,000.

$$f(x) = 1038(1.000134)^{-x}$$

5. What is the pressure at sea level?

6. The McDonald Observatory in Texas is at an altitude of 2000 meters. What is the approximate atmospheric pressure there?

7. As altitude increases, what happens to atmospheric pressure?

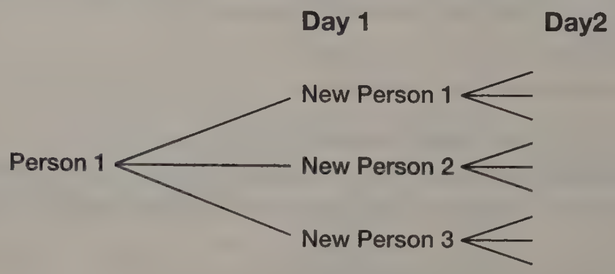
NAME ________________________________ DATE ____________ PERIOD ______

9-5 Enrichment

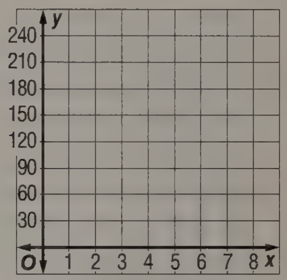
Pay it Forward

The idea behind "pay it forward" is that on the first day, one person does a good deed for three different people. Then, on the second day, those three people will each perform good deeds for 3 more people, so that on Day 2, there are 3 × 3 or 9 good deeds being done. Continue this process to fill in the chart. A tree diagram will help you fill in the chart.

Day	# of Deeds
0	1
1	3
2	9
3	
4	
5	



1. Graph the data you found in the chart as ordered pairs and connect with a smooth curve.



2. What type of function is your graph from problem 1? Write an equation that can be used to determine the number of good deeds on any given day, x.

3. How many good deeds will be performed on Day 21?

The formula, $\frac{3^{n+1}-3}{2}$, can be used to determine the **total** number of good deeds that have been performed, where n represents the day. For example, on Day 2, there have been 3 + 9 or 12 good deeds performed. Using the formula, you get, $\frac{3^{2+1}-3}{2}$ or $\frac{3^3-3}{2}$ + $\frac{27-3}{2}$ or a total of 12 good deeds performed.

4. Use this formula to determine the approximate number of good deeds that have been performed through Day 21.

5. Look up the world population. How does your number from Exercise 4 compare to the world population?

9-5 Spreadsheet Activity

Compound Interest

Banks often use spreadsheets to calculate and store financial data. One application is to calculate compound interest on an account.

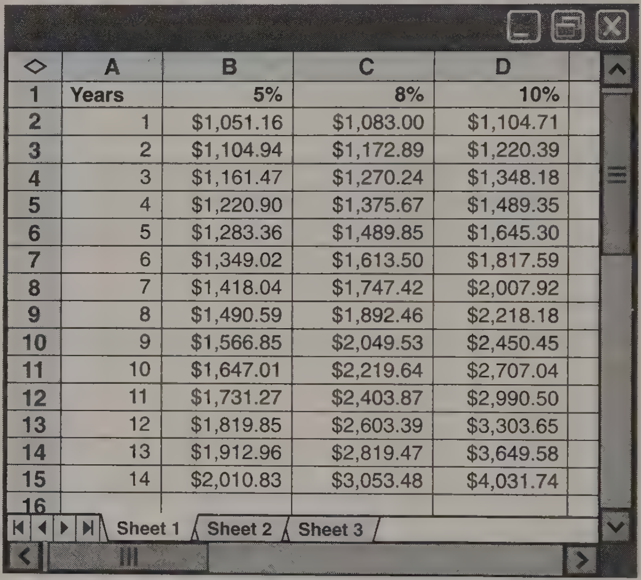
Example **Use a spreadsheet to find the time it will take an investment of $1000 to double. Suppose you can choose from investments that have annual interest rates of 5%, 8%, or 10% compounded monthly.**

The compound interest equation is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principal or initial investment, A is the final amount of the investment, r is the annual interest rate, n is the number of times interest is paid, or compounded, each year, and t is the number of years. In this case, $P = 1000$ and $n = 12$.

Step 1 Use Column A of the spreadsheet for the years.

Step 2 Columns B, C, and D contain the formulas for the final amounts of the investments. Format the cells in these columns as currency so that the amounts are shown in dollars and cents. Each formula will use the values in Column A as the value of t. For example, in cell C3, the formula is 1000 * (1 + (0.08/12))^(12 * A3).

Study the spreadsheet for the times when each investment exceeds $2000. At 5%, the $1000 will double in 14 years, at 8% it will double in 9 years, and at 10% it will double in 7 years.



◇	A	B	C	D
1	Years	5%	8%	10%
2	1	$1,051.16	$1,083.00	$1,104.71
3	2	$1,104.94	$1,172.89	$1,220.39
4	3	$1,161.47	$1,270.24	$1,348.18
5	4	$1,220.90	$1,375.67	$1,489.35
6	5	$1,283.36	$1,489.85	$1,645.30
7	6	$1,349.02	$1,613.50	$1,817.59
8	7	$1,418.04	$1,747.42	$2,007.92
9	8	$1,490.59	$1,892.46	$2,218.18
10	9	$1,566.85	$2,049.53	$2,450.45
11	10	$1,647.01	$2,219.64	$2,707.04
12	11	$1,731.27	$2,403.87	$2,990.50
13	12	$1,819.85	$2,603.39	$3,303.65
14	13	$1,912.96	$2,819.47	$3,649.58
15	14	$2,010.83	$3,053.48	$4,031.74
16				

Exercises

Use the spreadsheet of accounts involving compound interest.

1. How are the doubling times affected if the accounts compound interest quarterly instead of monthly?

2. How long will it take each account to reach $4000 if the interest is compounded monthly? quarterly?

3. How do the interest rate and the number of times the interest is compounded affect the growth of an investment?

NAME ______________________________ DATE______________ PERIOD _____

9-6 Lesson Reading Guide

Growth and Decay

Get Ready for the Lesson

Read the introduction to Lesson 9-6 in your textbook.

Suppose you want to predict the restaurant industry sales in the year 2009. What number should be substituted for t?

Read the Lesson

Match an equation to each situation, and then indicate whether the situation is an example of exponential growth or decay.

1. A coin had a value of $1.17 in 1995. Its value has been increasing at a rate of 9% per year.

A. $y = 1.17(1.09)^t$ **B.** $y = 1.17(0.91)^t$

2. A business owner has just paid $6000 for a computer. It depreciates at a rate of 22% per year. How much will it be worth in 5 years?

A. $A = 6000(1.22)^5$ **B.** $A = 6000(0.78)^5$

3. A city had a population of 14,358 residents in 2002. Since then, its population has been decreasing at a rate of about 5.5% per year.

A. $A = 14{,}358(1.055)^t$ **B.** $A = 14{,}358(0.945)^t$

4. Gina deposited $1500 in an account that pays 4% interest compounded quarterly. What will be the worth of the account in 2 years if she makes no deposits and no withdrawals?

A. $A = 1500(1.02)^2$ **B.** $A = 1500(1.01)^8$

Remember What You Learned

5. How can you use what you know about raising a number to the 0 power to help you remember what C represents in the exponential growth equation $A = C(1 + r)^t$ and the exponential decay equation $A = C(1 - r)^t$?

NAME ______________________________ DATE ____________ PERIOD _____

9-6 Study Guide and Intervention

Growth and Decay

Exponential Growth Population increases and growth of monetary investments are examples of **exponential growth**. This means that an initial amount increases at a steady rate over time.

Exponential Growth	The general equation for exponential growth is $y = C(1 + r)^t$. • y represents the final amount. • C represents the initial amount. • r represents the rate of change expressed as a decimal. • t represents time.

Example 1 **POPULATION The population of Johnson City in 2000 was 25,000. Since then, the population has grown at an average rate of 3.2% each year.**

a. Write an equation to represent the population of Johnson City since 2000.

The rate 3.2% can be written as 0.032.

$y = C(1 + r)^t$
$y = 25{,}000(1 + 0.032)^t$
$y = 25{,}000(1.032)^t$

b. According to the equation, what will the population of Johnson City be in the year 2010?

In 2010, t will equal 2010 − 2000 or 10.

Substitute 10 for t in the equation from part a.

$y = 25{,}000(1.032)^{10}$ $\quad t = 10$
$\approx 34{,}256$

In 2010, the population of Johnson City will be about 34,256.

Example 2 **INVESTMENT The Garcias have $12,000 in a savings account. The bank pays 3.5% interest on savings accounts, compounded monthly. Find the balance in 3 years.**

The rate 3.5% can be written as 0.035.

The special equation for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A represents the balance, P is the initial amount, r represents the annual rate expressed as a decimal, n represents the number of times the interest is compounded each year, and t represents the number of years the money is invested.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 12{,}000\left(1 + \frac{0.035}{12}\right)^{36}$$

$$A \approx 12{,}000(1.00292)^{36}$$

$$A \approx 13{,}328.09$$

In three years, the balance of the account will be $13,326.49.

Exercises

1. **POPULATION** The population of the United States has been increasing at an average annual rate of 0.91%. If the population of the United States was about 297,411,400 in the year 2005, predict the U. S. population in the year 2009. **Source:** U. S. Census Bureau

2. **INVESTMENT** Determine the amount of an investment of $2500 if it is invested at in interest rate of 5.25% compounded monthly for 4 years.

3. **POPULATION** It is estimated that the population of the world is increasing at an average annual rate of 1.3%. If the population of the world was about 6,472,416,997 in the year 2005, predict the world population in the year 2012. **Source:** U. S. Census Bureau

4. **INVESTMENT** Determine the amount of an investment of $100,000 if it is invested at an interest rate of 5.2% compounded quarterly for 12 years.

9-6 Study Guide and Intervention *(continued)*

Growth and Decay

Exponential Decay Radioactive decay and depreciation are examples of **exponential decay**. This means that an initial amount decreases at a steady rate over a period of time.

Exponential Decay	The general equation for exponential decay is $y = C(1 - r)^t$. • y represents the final amount. • C represents the initial amount. • r represents the rate of decay expressed as a decimal. • t represents time.

Example DEPRECIATION **The original price of a tractor was $45,000. The value of the tractor decreases at a steady rate of 12% per year.**

a. Write an equation to represent the value of the tractor since it was purchased.

The rate 12% can be written as 0.12.

$y = C(1 - r)^t$ General equation for exponential decay

$y = 45{,}000(1 - 0.12)^t$ $C = 45{,}000$ and $r = 0.12$

$y = 45{,}000(0.88)^t$ Simplify.

b. What is the value of the tractor in 5 years?

$y = 45{,}000(0.88)^t$ Equation for decay from part a

$y = 45{,}000(0.88)^5$ $t = 5$

$y \approx 23{,}747.94$ Use a calculator.

In 5 years, the tractor will be worth about $23,747.94.

Exercises

1. **POPULATION** The population of Bulgaria has been decreasing at an annual rate of 0.89%. If the population of Bulgaria was about 7,450,349 in the year 2005, predict its population in the year 2015. **Source:** U. S. Census Bureau

2. **DEPRECIATION** Carl Gossell is a machinist. He bought some new machinery for about $125,000. He wants to calculate the value of the machinery over the next 10 years for tax purposes. If the machinery depreciates at the rate of 15% per year, what is the value of the machinery (to the nearest $100) at the end of 10 years?

3. **ARCHAEOLOGY** The *half-life* of a radioactive element is defined as the time that it takes for one-half a quantity of the element to decay. Radioactive Carbon-14 is found in all living organisms and has a half-life of 5730 years. Consider a living organism with an original concentration of Carbon-14 of 100 grams.

   a. If the organism lived 5730 years ago, what is the concentration of Carbon-14 today?

   b. If the organism lived 11,460 years ago, determine the concentration of Carbon-14 today.

4. **DEPRECIATION** A new car costs $32,000. It is expected to depreciate 12% each year for 4 years and then depreciate 8% each year thereafter. Find the value of the car in 6 years.

NAME ______________________________ DATE ____________ PERIOD _____

9-6 Skills Practice

Growth and Decay

POPULATION For Exercises 1 and 2, use the following information.

The population of New York City increased from 8,008,278 in 2000 to 8,168,388 in 2005. The annual rate of population increase for the period was about 0.4%. **Source:** www.nyc.gov

1. Write an equation for the population t years after 2000.

2. Use the equation to predict the population of New York City in 2015.

SAVINGS For Exercises 3 and 4, use the following information.

The Fresh and Green Company has a savings plan for its employees. If an employee makes an initial contribution of $1000, the company pays 8% interest compounded quarterly.

3. If an employee participating in the plan withdraws the balance of the account after 5 years, how much will be in the account?

4. If an employee participating in the plan withdraws the balance of the account after 35 years, how much will be in the account?

5. **HOUSING** Mr. and Mrs. Boyce bought a house for $96,000 in 1995. The real estate broker indicated that houses in their area were appreciating at an average annual rate of 4%. If the appreciation remained steady at this rate, what was the value of the Boyce's home in 2005?

MANUFACTURING For Exercises 6 and 7, use the following information.

Zeller Industries bought a piece of weaving equipment for $60,000. It is expected to depreciate at an average rate of 10% per year.

6. Write an equation for the value of the piece of equipment after t years.

7. Find the value of the piece of equipment after 6 years.

8. **FINANCES** Kyle saved $500 from a summer job. He plans to spend 10% of his savings each week on various forms of entertainment. At this rate, how much will Kyle have left after 15 weeks?

9. **TRANSPORTATION** Tiffany's mother bought a car for $9000 five years ago. She wants to sell it to Tiffany based on a 15% annual rate of depreciation. At this rate, how much will Tiffany pay for the car?

9-6 Practice

Growth and Decay

COMMUNICATIONS For Exercises 1 and 2, use the following information.
Sports radio stations numbered 220 in 1996.

1. Write an equation for the number of sports radio stations for t years after 1996.

2. If the trend continues, predict the number of sports radio stations in this format for the year 2010.

3. **INVESTMENTS** Determine the amount of an investment if \$500 is invested at an interest rate of 4.25% compounded quarterly for 12 years.

4. **INVESTMENTS** Determine the amount of an investment if \$300 is invested at an interest rate of 6.75% compounded semiannually for 20 years.

5. **HOUSING** The Greens bought a condominium for \$110,000 in 2005. If its value appreciates at an average rate of 6% per year, what will the value be in 2010?

DEFORESTATION For Exercises 6 and 7, use the following information.
During the 1990s, the forested area of Guatemala decreased at an average rate of 1.7%.

6. If the forested area in Guatemala in 1990 was about 34,400 square kilometers, write an equation for the forested area for t years after 1990.

7. If this trend continues, predict the forested area in 2015.

8. **BUSINESS** A piece of machinery valued at \$25,000 depreciates at a steady rate of 10% yearly. What will the value of the piece of machinery be after 7 years?

9. **TRANSPORTATION** A new car costs \$18,000. It is expected to depreciate at an average rate of 12% per year. Find the value of the car in 8 years.

10. **POPULATION** The population of Osaka, Japan declined at an average annual rate of 0.05% for the five years between 1995 and 2000. If the population of Osaka was 11,013,000 in 2000 and it continues to decline at the same rate, predict the population in 2050.

NAME ______________________ DATE ____________ PERIOD ______

9-6 Word Problem Practice

Growth and Decay

1. **DEPRECIATION** The value of a new plasma television depreciates by about 7% each year. Aeryn purchases a 50-inch plasma television for $5000. What is its value after 4 years? Round your answer to the nearest hundred.

2. **MONEY** Hans opens a savings account by depositing $1200 in an account that earns 3 percent interest compounded weekly. How much will his investment be worth in 10 years? Assume that there are exactly 52 weeks in a year and round your answer to the nearest cent.

3. **HIGHER EDUCATION** The table lists the average costs of attending a four-year college in the United States during the 2005–2006 school years.

College Sector	Tuition and Fees	Room and Board
Four-year Public	$5,491	$6,636
Four-year Private	$21,235	$7,791

Source: www.collegeboard.com

Russ's parents invested money in a savings account earning an average of 4.5 percent interest, compounded monthly. After 15 years, they have exactly the right amount to cover the tuition, fees, room and board for Russ's first year at a public college. What was their initial investment? Round your answer to the nearest dollar.

4. **POPULATION** In the 2000 U.S. Census, the population of the United States was estimated at 281 million. The annual rate of growth is about 1.1%. At this rate, what is the expected population at the time of the 2020 census? Round your answer to the nearest ten million.

MEDICINE For Exercises 5–7, use the following information. If necessary, round your answers to the nearest tenth.

When doctors prescribe medication, they have to consider the rate at which the body filters a drug from the bloodstream. Suppose it takes the human body 6 days to filter out half of the Flu-B-Gone vaccine. The amount of Flu-B-Gone vaccine remaining in the bloodstream x hours after an injection is given by the equation $y = y_0(0.5)^{\frac{x}{6}}$, where y_0 is the initial amount. Suppose a doctor injects a patient with 20 μg (micrograms) of Flu-B-Gone.

5. How much of the vaccine will remain after 1 day?

6. How much of the vaccine will remain after 12 days?

7. After how many days will the amount of vaccine be less than 1 μg?

NAME ______________________ DATE ____________ PERIOD _____

9-6 Enrichment

Growth and Decay

Sierpinski Triangle is an example of a fractal that changes exponentially. Start with an equilateral triangle and find the midpoints of each side. Then connect the midpoints to form a smaller triangle. Remove this smaller triangle from the larger one.

Repeat the process to create the next triangle in the sequence. Find the midpoints of the sides of the three remaining triangles and connect them to form smaller triangles to be removed.

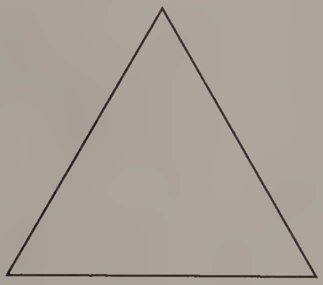


figure 1

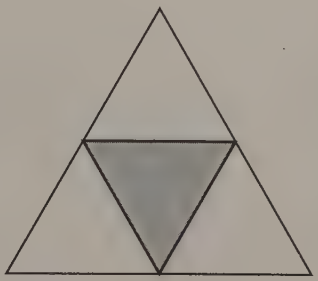


figure 2

Cut out: $\frac{1}{4}$

Area $= 1 - \frac{1}{4}$ or $\frac{3}{4}$

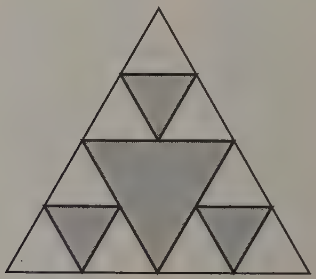


figure 3

Cut out: $\frac{1}{4} + \frac{3}{16}$ or $\frac{7}{16}$

Area $= 1 - \frac{7}{16}$ or $\frac{9}{16}$

1. Find the next triangle in the sequence. How much has been cut out? What is the area of the fourth figure in the sequence?

2. Make a conjecture as to what you need to multiply the previous amount cut by to find the new amount cut.

3. Fill in the chart to represent the amount cut and the area remaining for each triangle in the sequence.

figure	1	2	3	4	5	6
amount cut	0	$\frac{1}{4}$	$\frac{7}{16}$			
area remaining	1	$\frac{3}{4}$	$\frac{9}{16}$			

4. Write an equation to represent the area that is left in the nth triangle in the sequence.

5. If this process is continued, make a conjecture as to the remaining area.

Student Recording Sheet

Read each question. Then fill in the correct answer.

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Ⓕ Ⓖ Ⓗ Ⓙ

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Ⓕ Ⓖ Ⓗ Ⓙ

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Ⓕ Ⓖ Ⓗ Ⓙ

9. Record your answer and fill in the bubbles in the grid below. Be sure to use the correct place value.

				.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	①		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	⑤		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

10. Ⓐ Ⓑ Ⓒ Ⓓ

11. Ⓕ Ⓖ Ⓗ Ⓙ

Pre-AP

Record your answers for Question 12 on the back of this paper.

Assessment

NAME ______________________________ DATE ____________ PERIOD _____

9 Rubric for Scoring Pre-AP

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended-response questions require the student to show work.
- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is *not* considered a fully correct response.
- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 12 Rubric

Score	Specific Criteria
4	A correct solution that is supported by well-developed, accurate explanations. Each step in solving the equations are shown. The ball was in the air for approximately 5.01 seconds. The ball was 101.0 foot high at its highest point.
3	A generally correct solution, but may contain minor flaws in reasoning or computation.
2	A partially correct interpretation and/or solution to the problem.
1	A correct solution with no evidence or explanation.
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given.

NAME ______________________________ DATE ____________ PERIOD ______

9

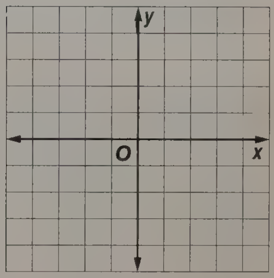
Chapter 9 Quiz 1

SCORE ______

(Lessons 9-1 and 9-2)

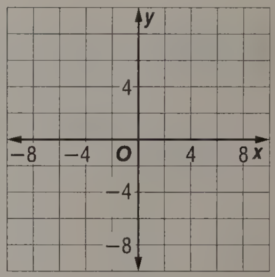
1. Use a table of values to graph $y = 2x^2 - 8x + 4$.

1.



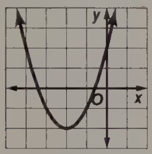
2. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $y = -x^2 - 4x + 5$. Identify the vertex as a maximum or a minimum. Then graph the function.

2.

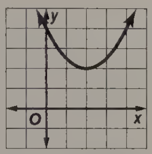


3. MULTIPLE CHOICE Which graph corresponds with the equation $y + 2 = (x - 2)^2$?

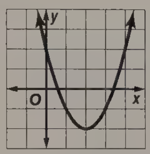
A.



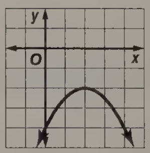
B.



C.



D.



3. ____________________

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

4. $x^2 - 2x - 24 = 0$

5. $3m^2 - 6m + 1 = 0$

4. ____________________

5. ____________________

NAME ______________________________ DATE ____________ PERIOD ______

9

Chapter 9 Quiz 2

SCORE ______

(Lesson 9-3)

Solve by taking the square root of each side. Round to the nearest tenth if necessary.

1. $r^2 - 10r + 25 = 12$

1. ____________________

2. $x^2 + 14x + 49 = 81$

2. ____________________

3. Find the value of c that makes $p^2 + 9p + c$ a perfect square.

3. ____________________

Solve each equation by completing the square. Round to the nearest tenth if necessary.

4. $w^2 + 8w - 10 = 10$

4. ____________________

5. $3x^2 - 2x = 27$

5. ____________________

NAME ____________________ DATE ________ PERIOD _____

SCORE _____

9

Chapter 9 Quiz 3

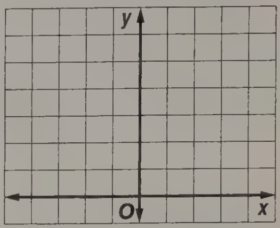
(Lessons 9-4 and 9-5)

1. Solve $4n^2 - 3n - 7 = 0$ by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. ________________

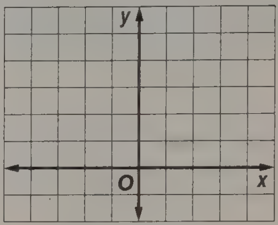
2. State the value of the discriminant for $3y^2 = y - 14$. Then determine the number of real roots of the equation.

2. ________________



3. Graph $y = \left(\frac{1}{3}\right)^x$. State the y-intercept. Then use the graph to determine the approximate value of $\left(\frac{1}{3}\right)^{-1.7}$.

3. ________________



4. Graph $y = 4(2^x) - 1$. State the y-intercept.

4. ________________

NAME ____________________ DATE ________ PERIOD _____

SCORE _____

9

Chapter 9 Quiz 4

(Lesson 9-6)

Robin has $1500 to invest. He invests the money at 4.85% compounded quarterly for 12 years.

1. Write an equation to represent the amount of money he will have in 12 years.

1. ________________

2. How much money will he have in 12 years?

2. ________________

A new car costs $35,000. It is expected to depreciate 8% each year.

3. Write an equation to represent the value of the car in t years.

3. ________________

4. Find the value of the car in 7 years.

4. ________________

5. A city whose population is 850,000 is growing at an average rate of about .2% per year. If the trend continues, what will the population be in 10 years?

5. ________________

NAME ______________________________ DATE ____________ PERIOD ______

9 Chapter 9 Mid-Chapter Test

SCORE ______

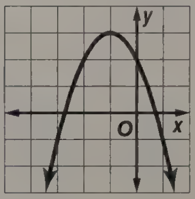
(Lessons 9-1 through 9-4)

Part I *Write the letter for the correct answer in the blank at the right of each question.*

1. Which equation corresponds to the graph shown?

A. $y = x^2 - 2x + 2$ **C.** $y = x^2 + 2x + 2$

B. $y = -x^2 + 2x + 2$ **D.** $y = -x^2 - 2x + 2$



1. ______

2. Find the coordinates of the vertex of the graph of $y = x^2 - 8x + 10$. Identify the vertex as a maximum or a minimum.

F. (4, −6); minimum **H.** (4, 6); maximum

G. (−4, 58); maximum **J.** (−4, 26); minimum 2. ______

3. Solve $x^2 - 24x + 144 = 36$ by taking the square root of each side.

A. −6, 18 **B.** 6, 18 **C.** 6, 12 **D.** −6, 6 3. ______

4. Which equation can be used to solve $5b^2 + 30b - 10 = 0$ by completing the square?

F. $(b + 6)^2 = 38$ **G.** $(b + 6)^2 = 46$ **H.** $(b + 3)^2 = 11$ **J.** $(b + 3)^2 = 19$ 4. ______

5. Which step is *not* performed in the process of solving $r^2 + 8r + 5 = 0$ by completing the square?

A. Subtract 5 from each side. **C.** Add 16 to each side.

B. Factor $r^2 + 8r$. **D.** Take the square root of each side. 5. ______

Part II

Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

6. $x^2 - 7x - 8 = 0$ 6. ______________

7. $x^2 + 1 = 5x$ 7. ______________

Solve each equation by completing the square. Round to the nearest tenth if necessary.

8. $-2x^2 + 18 = 7x$ 8. ______________

9. $x^2 + 4x = 20$ 9. ______________

10. The base of a rectangle is 4 more than the height. The area of the rectangle is 15 square inches. What are the dimensions of the rectangle? 10. ______________

NAME ______________________ DATE ____________ PERIOD ______

SCORE ______

9 Chapter 9 Vocabulary Test

- axis of symmetry
- completing the square
- compound interest
- discriminant
- exponential function
- general equation for exponential decay
- general equation for exponential growth
- maximum
- minimum
- parabola
- quadratic equation
- Quadratic Formula
- quadratic function
- roots
- symmetry
- vertex
- zeros

Choose a term from the vocabulary list above to complete each sentence.

1. You need to find the values of x for which $f(x) = 0$ to find the ___?___ of a function. **1.** ______________

2. Symmetry is a geometric property of a ___?___. **2.** ______________

3. You can use the discriminant to determine the number of ___?___ of a quadratic equation. **3.** ______________

4. The formula for compound interest is a special case of ___?___. **4.** ______________

5. The ___?___ is the result of solving the standard form of the quadratic equation for x. **5.** ______________

6. The ___?___ of a parabola is a minimum or maximum point. **6.** ______________

7. The equation $y = C(1 - r)^t$ is the ___?___. **7.** ______________

8. The graph of a(n) ___?___ is a parabola. **8.** ______________

9. Completing the square is a method you can use to solve a(n) ___?___. **9.** ______________

10. $y = 2^x$ is an example of a(n) ___?___. **10.** ______________

Define each term in your own words.

11. axis of symmetry **11.** ______________

12. discriminant **12.** ______________

NAME ______________________________ DATE ____________ PERIOD ______

9 Chapter 9 Test, Form 1

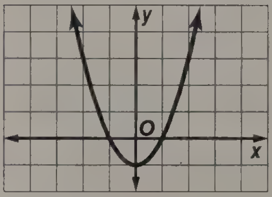
SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Which equation corresponds to the graph shown?

A. $y = x^2 + 1$ **C.** $y = x^2 - 1$

B. $y = -x^2 - 1$ **D.** $y = x^2$



1. ______

2. What is the equation of the axis of symmetry of the graph of $y = x^2 + 6x - 7$?

F. $x = 6$ **H.** $x = 3$

G. $x = -3$ **J.** $x = -6$

2. ______

3. Find the coordinates of the vertex of the graph of $y = 4 - x^2$. Identify the vertex as a maximum or a minimum.

A. (2, 0); maximum **C.** (0, 4); maximum

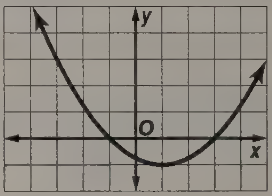
B. (0, 4); minimum **D.** (2, 0); minimum

3. ______

4. What are the roots of the quadratic equation whose related function is graphed at the right?

F. −1, 3 **H.** −3, 1

G. −1, 1 **J.** 1, 3

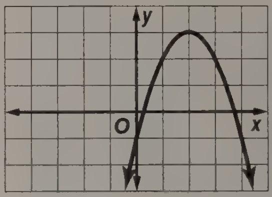


4. ______

5. One root of the quadratic equation whose related function is graphed lies between which consecutive integers?

A. 1 and 2 **C.** 0 and −1

B. 2 and 3 **D.** 0 and 1



5. ______

6. Solve $k^2 + 4k + 4 = 25$ by taking the square root of each side.

F. −5, 5 **G.** 5 **H.** −3, 7 **J.** −7, 3

6. ______

7. Which value of c makes $y^2 + 8y + c$ a perfect square?

A. 4 **B.** 16 **C.** 64 **D.** 8

7. ______

8. Which equation is equivalent to $x^2 + 2x - 3 = 0$?

F. $(x + 1)^2 = 2$ **G.** $(x - 1)^2 = 4$ **H.** $(x - 1)^2 = 2$ **J.** $(x + 1)^2 = 4$

8. ______

9. Solve the equation $2x^2 + 3x - 5 = 0$ by using the Quadratic Formula.

A. $-2\frac{1}{2}, 1$ **B.** −5, 1 **C.** $-1, 2\frac{1}{2}$ **D.** −1, 5

9. ______

10. State the value of the discriminant for $y = x^2 - 8x + 10$.

F. 4.9 **G.** 24 **H.** 104 **J.** 10.2

10. ______

NAME ______________________ DATE __________ PERIOD _____

9 Chapter 9 Test, Form 1 *(continued)*

11. Determine the number of real roots of $n^2 - 5n - 6 = 0$.

A. 1 real root **C.** infinitely many real roots

B. 2 real roots **D.** no real roots **11.** _____

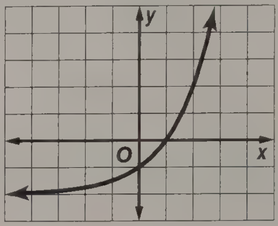
12. Which function is quadratic?

F. $y = x(3x + 2)$ **G.** $x - 5y = 9$ **H.** $y = 5^x - 2$ **J.** $y = x^3 + 2x^2$ **12.** _____

13. Which equation corresponds to the graph shown?

A. $y = 2^x + 2$ **C.** $y = 2^x - 2$

B. $y = \left(\frac{1}{2}\right)^x - 2$ **D.** $y = \left(\frac{1}{2}\right)^x + 2$ **13.** _____



14. What is the y-intercept on the graph shown?

F. $\frac{1}{2}$ **G.** 1 **H.** 0 **J.** -1 **14.** _____

15. TOURNAMENTS A chess tournament starts with 16 people competing. The exponential function $y = 16\left(\frac{1}{2}\right)^x$ describes how many people are remaining in the tournament after x rounds. How many people are left in the tournament after 2 rounds?

A. 4 **B.** 2 **C.** 8 **D.** 1 **15.** _____

16. Which set of data displays exponential behavior?

F.

x	0	1	2	3
y	0	2	4	6

H.

x	0	1	2	3
y	1	10	100	1,000

G.

x	0	1	2	3
y	0	1	16	81

J.

x	0	1	2	3
y	6	4	2	1

16. _____

17. INVESTMENTS Determine the amount of an investment if $1000 is invested at an interest rate of 8% compounded quarterly for 2 years.

A. $1160.00 **B.** $1171.66 **C.** $1040.40 **D.** $1166.40 **17.** _____

18. Which equation represents exponential decay?

F. $y = 0.5(1.2)^x$ **G.** $y = 10(1.1)^x$ **H.** $y = 100(0.9)^x$ **J.** $y = 0.1x^2$ **18.** _____

19. BIOLOGY If $y = 10(2.5)^t$ represents the number of bacteria in a culture at time t, how many will there be at time $t = 6$?

A. 2441 **B.** 244 **C.** 24 **D.** none **19.** _____

20. A $60,000 piece of machinery depreciates in value at a rate of 11% per year. About what will its value be in 5 years?

F. $47,526 **G.** $42,298 **H.** $33,504 **J.** $37,645 **20.** _____

Bonus If $b^2 - 4ac = 0$, determine the number of real roots of the equation $ax^2 + bx + c = 0$. **B:** __________

NAME ______________________ DATE __________ PERIOD _____

9 Chapter 9 Test, Form 2A

SCORE _____

Write the letter for the correct answer in the blank at the right of each question.

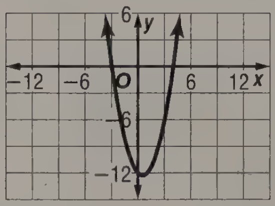
1. Which equation corresponds to the graph shown?

A. $y = x^2 - 7x - 12$

B. $y = x^2 - x - 12$

C. $y = x^2 + 5x + 12$

D. $y = x^2 + 12x - 1$



1. ______

2. Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of $y = 2x^2 - 12x + 6$.

F. $x = -3$; $(-3, 60)$ **H.** $x = -3$; $(-3, 78)$

G. $x = 3$; $(3, -12)$ **J.** $x = 3$; $(3, 6)$

2. ______

3. Find the coordinates of the vertex of the graph of $y = -2x^2 - 8$. Identify the vertex as a maximum or a minimum.

A. $(-2, -16)$; minimum **C.** $(2, -16)$; maximum

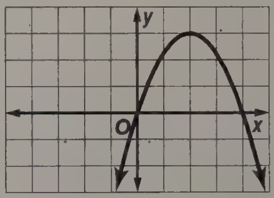
B. $(-2, 8)$; maximum **D.** $(0, -8)$; maximum

3. ______

4. What are the root(s) of the quadratic equation whose related function is graphed at the right?

F. 2 **H.** 0, 4

G. 3 **J.** −4, 0

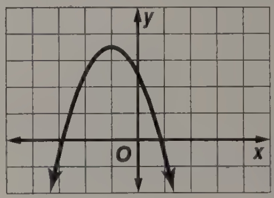


4. ______

5. One root of the quadratic equation whose related function is graphed lies between which consecutive integers?

A. −3 and −2 **C.** −2 and −1

B. 2 and 3 **D.** 1 and 2



5. ______

6. Solve $a^2 - 22a + 121 = 16$ by taking the square root of each side.

F. 4 **G.** 7, 15 **H.** ±4 **J.** −15, −7

6. ______

7. What value of c makes $3x^2 + 24x + c$ contain a perfect square trinomial?

A. 144 **B.** 16 **C.** 64 **D.** 48

7. ______

8. Which step is *not* performed in the process of solving $n^2 - 12n - 10 = 0$ by completing the square?

F. Add 10 to each side. **H.** Factor $n^2 - 12n - 10 = 0$.

G. Add 36 to each side. **J.** Take the square root of each side.

8. ______

9. Which equation is equivalent to $2x^2 - 24x - 14 = 0$?

A. $(x - 6)^2 = 50$ **B.** $(x - 3)^2 = 13$ **C.** $(x - 3)^2 = 20$ **D.** $(x - 6)^2 = 43$

9. ______

NAME ______________________ DATE ____________ PERIOD ______

9 Chapter 9 Test, Form 2A *(continued)*

For Questions 10 and 11, solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

10. $4x^2 + 11x - 3 = 0$

F. $-2.4, -0.3$ **G.** $-\frac{1}{4}, 3$ **H.** $0.3, 2.4$ **J.** $-3, \frac{1}{4}$ **10.** ______

11. $y^2 + 8y = 2$

A. $-8.2, 0.2$ **B.** $8.2, -0.2$ **C.** $0.3, 7.7$ **D.** $-7.7, -0.3$ **11.** ______

12. Determine the number of real roots of $7x^2 - 18x + 12 = 0$.

F. 2 **G.** infinitely many **H.** none **J.** 1 **12.** ______

13. State the value of the discriminant for $4x^2 + 5x = 6$.

A. 121 **B.** 101 **C.** -71 **D.** 11 **13.** ______

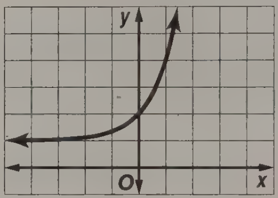
14. Which equation represents exponential growth?

F. $y = 5(.84)^x$ **G.** $y = 5x$ **H.** $y = 0.3x^3$ **J.** $y = 5(1.06)^x$ **14.** ______

15. Which equation corresponds to the graph shown?

A. $y = (3)^x + 1$ **C.** $y = 2(3^x)$

B. $y = 2(3^x + 1)$ **D** $(2 \cdot 3)^x + 1$ **15.** ______



16. A weight lifter can deadlift 315 pounds. She can increase the weight $W(x)$ that she can lift according to the function $W(x) = 315(1.05)^x$, where x represents the number of training cycles completed. How much will she deadlift after 4 training cycles?

F. 365 lb **G.** 383 lb **H.** 378 lb **J.** 402 lb **16.** ______

17. Which set of data displays exponential behavior?

A.

x	0	1	2	3
y	1	7	49	343

C.

x	0	1	2	3
y	0	9	18	27

B.

x	0	1	2	3
y	0	1	4	27

D.

x	0	1	2	3
y	24	12	6	1

17. ______

18. **BIOLOGY** A certain fast-growing bacteria increases 6% per minute. If there are 100 bacteria now, about how many will there be 12 minutes later?

F. 172 **G.** 201 **H.** 48 **J.** 190 **18.** ______

19. **POPULATION** A city's population is about 954,000 and is decreasing at an annual rate of 0.1%. Predict the population in 50 years.

A. 577,176 **B.** 906,300 **C.** 1,002,888 **D.** 907,450 **19.** ______

20. Anna deposited $2500 at 3.5% per year compounded monthly. How much money will she have in 5 years?

F. $14,030.81 **G.** $4,156.90 **H.** $2,977.36 **J.** $8,988.60 **20.** ______

Bonus What is the equation of the axis of symmetry of a parabola if its x-intercepts are -3 and 5? **B:** ______________

NAME ______________________ DATE __________ PERIOD ______

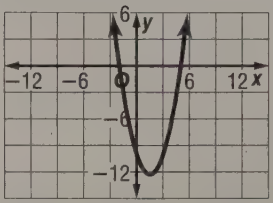
9 Chapter 9 Test, Form 2B

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Which equation corresponds to the graph shown?

A. $y = x^2 - 3x - 10$
B. $y = x^2 + 7x + 10$
C. $y = x^2 - 10x + 6$
D. $y = x^2 - 11x - 10$



1. ______

2. Find the equation of the axis of symmetry and the coordinates of the vertex of the graph of $y = -x^2 - 10x + 17$.

F. $x = -5$; $(-5, -8)$
G. $x = -5$; $(-5, 42)$
H. $x = 5$; $(5, 92)$
J. $x = 5$; $(5, 32)$

2. ______

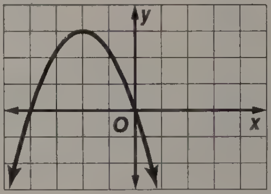
3. Find the coordinates of the vertex of the graph of $y = 3x^2 - 6$. Identify the vertex as a maximum or a minimum.

A. $(0, -6)$; maximum
B. $(-6, 0)$; minimum
C. $(0, -6)$; minimum
D. $(6, 0)$; minimum

3. ______

4. What are the root(s) of the quadratic equation whose related function is graphed at the right?

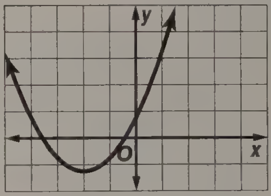
F. 4, 0
G. −2, 3
H. 2, 3
J. −4, 0



4. ______

5. One root of the quadratic equation whose related function is graphed lies between which consecutive integers?

A. −4 and −3
B. 0 and 1
C. −5 and −4
D. −3 and −2



5. ______

6. Solve $b^2 - 16b + 64 = 9$ by taking the square root of each side.

F. −5, −11
G. 17
H. ±17
J. 5, 11

6. ______

7. What value of c makes $4x^2 + 24x + c$ a perfect square trinomial?

A. 1
B. 36
C. 144
D. 9

7. ______

8. Which step is *not* performed in the process of solving $r^2 + 12r - 6 = 0$ by completing the square?

F. Add 6 to each side.
G. Factor $r^2 + 12r - 6$.
H. Add 36 to each side.
J. Take the square root of each side.

8. ______

9. Which equation is equivalent to $3x^2 + 24x + 15 = 0$?

A. $(x + 4)^2 = 11$
B. $(x + 4)^2 = 1$
C. $(x + 2)^2 = -1$
D. $(x + 2)^2 = -11$

9. ______

NAME ______________________________ DATE ____________ PERIOD ______

9 Chapter 9 Test, Form 2B *(continued)*

SCORE ______

For Questions 10 and 11, solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

10. $3x^2 - 11x - 4 = 0$

F. $-4, \frac{1}{3}$ **G.** 0.4, 3.3 **H.** $-\frac{1}{3}, 4$ **J.** −3.3, −0.4 **10.** ______

11. $d^2 + 4 = 6d$

A. −5.2, −0.8 **B.** −6.6, 0.6 **C.** −0.6, 6.6 **D.** 0.8, 5.2 **11.** ______

12. Determine the number of real roots of $6x^2 + 19x + 14 = 0$.

F. 2 **G.** infinitely many **H.** 1 **J.** none **12.** ______

13. State the value of the discriminant for $3b^2 + 6b = 10$.

A. −84 **B.** 156 **C.** 126 **D.** 12.5 **13.** ______

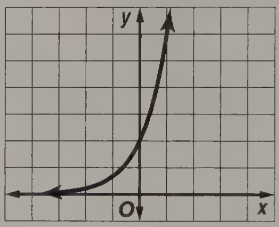
14. Which equation represents exponential decay?

F. $y = .5x^3$ **G.** $y = .5x^2 - x$. **H.** $y = .5(1.07)^x$ **J.** $y = .5(.87)^x$ **14.** ______

15. Which equation corresponds to the graph shown?

A. $y = 3^x + 2$ **C.** $y = 2(3^x)$

B. $y = 2(3^x + 1)$ **D.** $y = (2 \cdot 3)^x + 1$ **15.** ______



16. A weight lifter can deadlift 275 pounds. She can increase the weight $W(x)$ that she can lift according to the function $W(x) = 275(1.05)^x$, where x represents the number of training cycles completed. How much will she deadlift after 5 training cycles?

F. 334 lb **G.** 369 lb **H.** 344 lb **J.** 351 lb **16.** ______

17. Which set of data displays exponential behavior?

A.

x	0	1	2	3
y	0	1	32	243

B.

x	0	1	2	3
y	1	5	25	125

C.

x	0	1	2	3
y	0	5	10	15

D.

x	0	1	2	3
y	20	10	5	1

17. ______

18. POPULATION A city's population is about 763,000 and is increasing at an annual rate of 1.5%. Predict the population of the city in 50 years.

F. 1,335,250 **G.** 826,830,628 **H.** 358,374 **J.** 1,606,300 **18.** ______

19. BUSINESS A printing press valued at $120,000 depreciates 12% per year. What will be the approximate value of the printing press in 7 years?

A. $19,200 **B.** $265,282 **C.** $49,041 **D.** $55,728 **19.** ______

20. Juan deposited $3500 at 4.5% per year compounded quarterly. How much money will he have in 8 years?

F. $8,980.25 **G.** $15,471.77 **H.** $5,006.58 **J.** $5,636.14 **20.** ______

Bonus Find all values of c that make $x^2 + cx + 64$ a perfect square. **B:** ______________

NAME ______________________________ DATE ____________ PERIOD ______

9

Chapter 9 Test, Form 2C

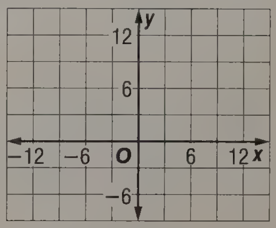
SCORE ______

Assessment

For Questions 1 and 2, use a table of values to graph each function.

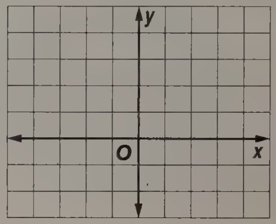
1. $y = -x^2 + 3x + 10$

1.



2. $y = 2x^2 - 3x$

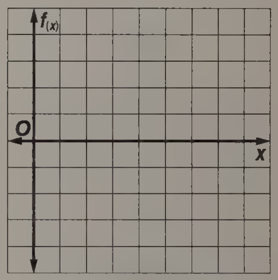
2.



3. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $y = 2x^2 - 8x + 3$. Identify the vertex as a maximum or a minimum.

3. ______________________

4. Solve $x^2 - 6x + 4 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.



4. ______________________

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

5. $t^2 - 2t + 1 = 10$

6. $w^2 + 20w + 100 = 169$

5. ______________________

6. ______________________

Solve each equation by completing the square. Round to the nearest tenth if necessary.

7. $2x^2 + 3x = 20$

8. $p^2 - 12p + 9 = 0$

7. ______________________

8. ______________________

For Questions 11 and 12, solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

9. $15n^2 - 3 = 4n$

10. $r^2 + 16r + 21 = 0$

9. ______________________

10. ______________________

9 Chapter 9 Test, Form 2C *(continued)*

For Questions 11 and 12, state the value of the discriminant for each equation. Then determine the number of real roots of the equation.

11. $7m^2 + 8m = 3$ **12.** $4p^2 = 4p - 1$

11. ________________

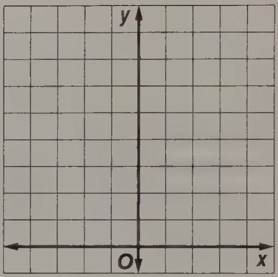
12. ________________

13. The length of a rectangle is 5 inches more than the width. The area is 33 square inches. Find the length and width. Round to nearest tenth if necessary.

13. ________________

14. A weight lifter can bench-press 165 pounds. She plans to increase the weight $W(x)$ in pounds that she is lifting according to the function $W(x) = 165(1.05)^x$, where x represents the number of training cycles she completes. How much will she bench-press after 4 training cycles?

14. ________________



15. Graph $y = 7^x$. State the y-intercept.

15. ________________

16. Use the graph of $y = 7^x$ to determine the approximate value of $7^{0.6}$.

16. ________________

For Questions 17 and 18, determine whether the data in each table display exponential behavior. Explain why or why not.

17.

x	1	2	3	4
y	−4	12	−48	192

17. ________________

18.

x	2	5	8	11	14
y	2	6	18	54	162

18. ________________

19. INVESTMENTS Determine the amount of an investment if \$700 is invested at an interest rate of 8% compounded monthly for 9 years.

19. ________________

20. BUSINESS A new welding machine valued at \$38,000 depreciates at a steady rate of 9% per year. What is the value of the welding machine in 10 years?

20. ________________

Bonus Without graphing, determine the x-intercepts of the graph of $f(x) = 7x^2 + 9x + 1$.

B: ________________

NAME ______________________ DATE __________ PERIOD ______

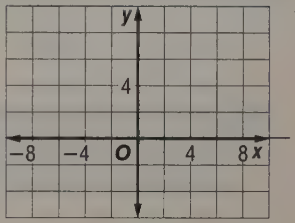
9 Chapter 9 Test, Form 2D

SCORE ______

For Questions 1 and 2, use a table of values to graph each function.

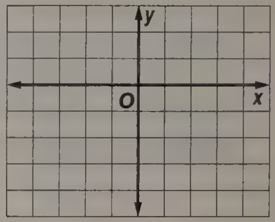
1. $y = x^2 - 7x + 12$

1.



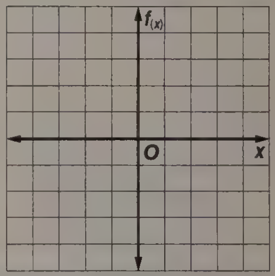
2. $y = 4x^2 - 8x$

2.



3. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $y = -2x^2 + 4x - 5$. Identify the vertex as a maximum or a minimum.

3. ______________



4. Solve $x^2 - 2x - 1 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

4. ______________

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

5. $m^2 - 18m + 81 = 100$

6. $r^2 + 10r + 25 = 15$

5. ______________

6. ______________

Solve each equation by completing the square. Round to the nearest tenth if necessary.

7. $2x^2 + 9x = 18$

8. $t^2 + 15t + 11 = 0$

7. ______________

8. ______________

For Questions 11 and 12, solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

9. $12v^2 - 6 = -v$

10. $d^2 - 14d + 22 = 0$

9. ______________

10. ______________

NAME ______________________________ DATE ______________ PERIOD ________

Chapter 9 Test, Form 2D *(continued)*

For Questions 11 and 12, state the value of the discriminant for each equation. Then determine the number of real roots of the equation.

11. $3b^2 + 10 = -8b$ **12.** $9a^2 = 6a - 1$

11. ________________

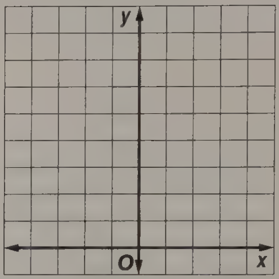
12. ________________

13. The width of a rectangle is 3 inches less than the length. The area is 50 square inches. Find the length and width. Round to nearest tenth if necessary.

13. ________________

14. A weight lifter can bench-press 145 pounds. She plans to increase the weight $W(x)$ in pounds that she is lifting according to the function $W(x) = 145(1.05)^x$, where x represents the number of training cycles she completes. How much will she bench-press after 5 training cycles?

14. ________________



15. Graph $y = \left(\frac{1}{6}\right)^x$. State the y-intercept.

15. ________________

16. Use the graph of $y = \left(\frac{1}{6}\right)^x$ to determine the approximate value of $\left(\frac{1}{6}\right)^{-0.7}$.

16. ________________

For Questions 17 and 18, determine whether the data in each table display exponential behavior. Explain why or why not.

17.

x	1	2	3	4
y	2	5	8	11

17. ________________

18.

x	−1	1	3	5	7
y	7	14	28	56	112

18. ________________

19. ART An oil painting originally cost \$2500 and increases in value at a rate of 6% per year. Find the value of the painting after 12 years.

19. ________________

20. CARS A new car valued at \$16,500 depreciates at a steady rate of 12% per year. What is the value of the car in 10 years?

20. ________________

Bonus Find the value of a so that the equation $ax^2 + 8x + 32 = 0$ has 1 real root.

B: ________________

NAME ______________________ DATE __________ PERIOD _____

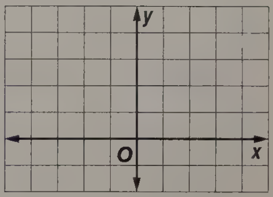
Chapter 9 Test, Form 3

SCORE _____

For Questions 1 and 2, use a table of values to graph each function.

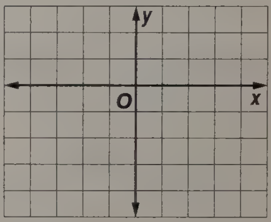
1. $y = -2x^2 - 3x + 3$

1.



2. $y = 3x^2 - 5x - 2$

2.



3. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $y - 1 = 5x^2 - 25x + 3$. Identify the vertex as a maximum or a minimum.

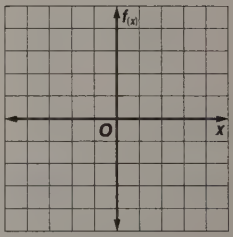
3. ______________________

4. Two points on a parabola are $(-3, 5)$ and $(11, 5)$. What is the equation of the axis of symmetry?

4. ______________________

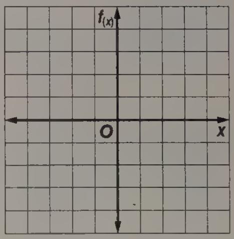
Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

5. $\frac{1}{2}x^2 + x - 4 = 0$



5. ______________________

6. $x^2 + 7 = 6x$



6. ______________________

For Questions 7 and 8, solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

7. $s^2 + 26s + 169 = 121$

8. $b^2 - 34b + 289 = 45$

9. Solve $3y^2 - 7y - 5 = 0$ by completing the square. Round to the nearest tenth if necessary.

10. Solve $x^2 + 10x + c = 0$ for x by completing the square.

7. ______________________

8. ______________________

9. ______________________

10. ______________________

9 Chapter 9 Test, Form 3 *(continued)*

11. Solve $21u^2 = u + 10$ by using the Quadratic Formula. Round to the nearest tenth if necessary.

11. ________________

12. Solve $-2x^2 = -(5x + 4)$ by using the Quadratic Formula. Round to nearest tenth if necessary.

12. ________________

13. PHYSICAL SCIENCE A projectile is shot vertically in the air from ground level. Its height h, in feet, after t seconds is given by $h = 88t - 16t^2$. Will the projectile have a height of 125 feet 0, 1, or 2 times after being shot?

13. ________________

14. Find the values for c so that $s^2 + cs + 81 = 0$ has one real root.

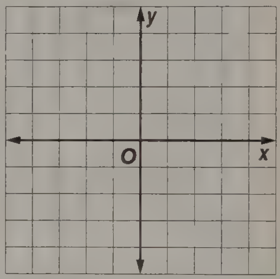
14. ________________

15. One number is 10 larger than another. Their product is 119. Find the numbers.

15. ________________

16. WEIGHT LIFTING A weight lifter can squat 475 pounds. She plans to increase the weight $W(x)$ in pounds that she is lifting according to the function $W(x) = 475(1.05)^x$, where x represents the number of training cycles she completes. How many training cycles must she complete to be able to squat 575 pounds?

16. ________________



17. Graph $y = 4(2^x - 1)$. State the y-intercept.

17. ________________

Determine whether the data in the table displays exponential behavior. Explain why or why not.

18.

x	0	1	2	3	4
y	10	20	40	80	160

18. ________________

19. INVESTMENTS Determine the amount of an investment if \$96,000 is invested at an interest rate of 5.2% compounded daily for 3 years.

19. ________________

20. Suppose a car that sells for \$40,000 depreciates 10% per year. How many years would it take for the car to have a value less than \$25,000?

20. ________________

Bonus Solve the equation $ax^2 + 7x - 4 = 0$ for x by using the Quadratic Formula.

B: ________________

NAME ______________________________ DATE __________ PERIOD _____

9 Chapter 9 Extended-Response Test

SCORE _____

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. Write a quadratic function whose graph opens upward. Write an exponential function. Graph both functions on the same coordinate plane. Compare and contrast the domains and ranges of the two functions and any symmetry of the two graphs.

2. **a.** Write a quadratic equation that has no real roots, and find its discriminant. Explain how the discriminant shows that a quadratic equation has no real roots.
   b. Write a quadratic equation that has one real root, and find its discriminant. Explain why the Quadratic Formula yields only one solution when the discriminant of a quadratic equation is equal to zero.
   c. Write a quadratic equation that has two real roots. Determine whether completing the square, graphing, or factoring would be the better method to use to solve your quadratic equation.

3. Honovi purchased a new car for $25,000 and has $5000 left to invest.
   a. Choose an interest rate between 4% and 7% for Honovi's investment, and find the length of time it would take for the investment to double.
   b. Choose an annual depreciation rate from 8% to 10% for the new car that Honovi purchased, and find the length of time it would take for the car's value to be equal to one-half of the purchase price.
   c. Using the rates from part **a** and part **b**, find the length of time it would take for the investment to be equal to the value of the car. What is the value at that time?

NAME ______________________ DATE ____________ PERIOD ________

SCORE ________

9

Standardized Test Practice

(Chapters 1–9)

Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. Armando collects baseball cards. He currently has 48 cards, and buys 5 new cards a week. How many cards will Armando have in 10 weeks? (Lesson 3-4)

A 102 **B** 98 **C** 94 **D** 100

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Wesley carves whistles out of wood and sells them at a gift shop. The equation $C = 2n + 15$ models his weekly cost C of making n whistles. The equation $R = 5n$ models his weekly revenue R from selling n whistles. How many whistles must Wesley make and sell for his cost and revenue to be equal? (Lesson 5-2)

F 5 **G** 4 **H** 25 **J** 10

2. Ⓕ Ⓖ Ⓗ Ⓙ

3. Solve $2(5x - 4) \geq 7(x - 2)$. (Lesson 6-3)

A $x \geq -2$ **B** $x \leq -2$ **C** $x \geq \frac{2}{3}$ **D** $x \geq 2$

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Find $3mn(2m^2 - 3mn + n^2)$. (Lesson 7-5)

F $5m^3n - 6m^2n + 3mn^3$ **H** $6m^3n - 9m^2n^2 + 3mn^3$

G $6m^2n - 3mn + 3mn^2$ **J** $2m^2 + n^2$

4. Ⓕ Ⓖ Ⓗ Ⓙ

5. Which binomial is a factor of $6ut - 9u + 8vt - 12v$? (Lesson 8-2)

A $2t + 3$ **B** $3u + 4$ **C** $-2t + 3$ **D** $3u + 4v$

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Factor $16 - n^4$. (Lesson 8-5)

F $(4 - n^2)(4 + n^2)$ **H** $(2 - n)(2 + n)(n^2 + 4)$

G $(n + 2)(n - 2)(4 + n^2)$ **J** $(2 - n)(2 + n)(2 - n)(2 + n)$

6. Ⓕ Ⓖ Ⓗ Ⓙ

7. Solve $2x^2 + 72 = 24x$. (Lesson 8-6)

A $\{2, 6\}$ **B** $\{-6, 2\}$ **C** $\{6\}$ **D** $\{-6\}$

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Find the coordinates of the vertex of the graph of $y = 3x^2 + 2x$. (Lesson 9-1)

F $\left(\frac{1}{3}, 1\right)$ **G** $\left(-\frac{1}{3}, -\frac{1}{3}\right)$ **H** $\left(\frac{1}{3}, -\frac{1}{3}\right)$ **J** $\left(-\frac{1}{3}, 1\right)$

8. Ⓕ Ⓖ Ⓗ Ⓙ

9. Which equation is equivalent to $2r^2 - 28r + 38 = 0$? (Lesson 9-3)

A $2(r - 7)^2 = 49$ **C** $(r - 14)^2 = 177$

B $(r - 14)^2 = 60$ **D** $(r - 7)^2 = 30$

9. Ⓐ Ⓑ Ⓒ Ⓓ

10. What number should be inserted into the table for the data to display exponential behavior? (Lesson 9-5)

x	2	3	4	5
y	8	24	72	

F 152 **G** 186 **H** 248 **J** 216

10. Ⓕ Ⓖ Ⓗ Ⓙ

NAME ______________________________ DATE ____________ PERIOD ______

9 Standardized Test Practice *(continued)*

SCORE ______

11. A watch is on sale for 30% off the original price. If the original price of the watch is $14, what is the discounted price? (Lesson 2-7)

A $9.80 **B** $4.20 **C** $13.70 **D** $9.33 **11.** Ⓐ Ⓑ Ⓒ Ⓓ

12. Solve the proportion $\frac{n}{500} = \frac{2}{40}$. (Lesson 2-6)

F 125 **G** 25 **H** 16 **J** 80 **12.** Ⓕ Ⓖ Ⓗ Ⓙ

13. Determine the x-intercept and y-intercept of $4x - 2y = 10$. (Lesson 3-3)

A 0, 0 **B** 2.5, 0 **C** 0, −5 **D** 2.5, −5 **13.** Ⓐ Ⓑ Ⓒ Ⓓ

14. Write a direct variation equation that relates x and y if $y = 10$ when $x = 12$. (Lesson 4-2)

F $y = \frac{6}{5}x$ **G** $y = \frac{5}{6}x$ **H** $y = 12x$ **J** $10y = 12x$ **14.** Ⓕ Ⓖ Ⓗ Ⓙ

15. Write the slope-intercept form of an equation for the line that passes through (4, 0) and is parallel to the graph of $3y - 6x = 4$. (Lesson 4-7)

A $2y = -x$ **B** $y = 2x - 8$ **C** $y = 2x + 4$ **D** $y = -2x + 8$ **15.** Ⓐ Ⓑ Ⓒ Ⓓ

16. Which ordered pair is not a solution of $4x - 8y \geq 24$? (Lesson 6-6)

F (−5, −4) **G** (2, −2) **H** (7, −1) **J** (−8, −8) **16.** Ⓕ Ⓖ Ⓗ Ⓙ

17. Which binomial is a factor of $6x^2 + x - 12$? (Lesson 8-4)

A $(3x - 4)$ **B** $(2x + 6)$ **C** $(3x + 4)$ **D** $(2x - 3)$ **17.** Ⓐ Ⓑ Ⓒ Ⓓ

Part 2: Griddable

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate circle that corresponds to that entry.

18. If $3x + 2y = 9$ and $3x + 3y = 6$, what is the value of x? (Lesson 5-3)

				.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	①		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	⑤		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

19. State the value of the discriminant for $4x^2 + 17x + 18 = 0$. (Lesson 9-4)

				.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	①		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	⑤		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

NAME ______________________ DATE ____________ PERIOD ______

9 Standardized Test Practice *(continued)*

(Chapters 1–9)

Part 3: Short Answer

Instructions: Write your answers in the space provided.

20. Solve $4y - 3(7y - 2) \le -14 - 13y$. (Lesson 6-3)

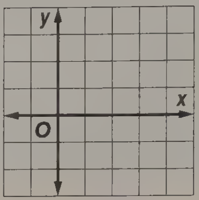
20. ______________

21. Solve the system of inequalities by graphing. (Lesson 6-7)

$x + y \le 4$

$y \ge 2x - 4$

21.



22. Arrange the terms of the polynomial $4x - 3 + 2x^2 + 3x^3$ so that the powers of x are in descending order. (Lesson 7-3)

22. ______________

23. Simplify $(4xy + 3x^2y - 5y^2) - (3y^2 - 5xy + 7x^2y)$. (Lesson 7-4)

23. ______________

24. Find $(3a^2 + 2)(3a^2 - 2)$. (Lesson 7-7)

24. ______________

25. Factor $x^2 + 12x + 35$. (Lesson 8-3)

25. ______________

26. Factor $2m^2 + 11m + 15$. (Lesson 8-4)

26. ______________

27. Solve $36 - \frac{1}{4}y^2 = 0$ by factoring. (Lesson 8-5)

27. ______________

28. Use the formula $h = -16t^2 + 250t$ to model the height h in feet of a model rocket t seconds after it is launched. Determine when the rocket will reach a height of 900 feet. (Lesson 9-4)

28. ______________

29. The population of North Carolina has been increasing at an annual rate of 1.7%. If the population of North Carolina was 7,650,789 in the year 1999, predict its population in 2010. (Lesson 9-6)

29. ______________

30. Consider $y = -x^2 - 2x + 2$. (Lesson 9-1)

a. Find the equation for the axis of symmetry.

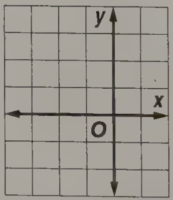
b. Find the coordinates of the vertex and determine if it is a maximum or minimum.

c. Graph the equation.

30a. ______________

30b. ______________

30c.

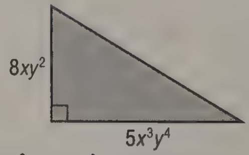


NAME ______________________ DATE ____________ PERIOD _____

Unit 3 Test

(Chapters 7–9)

1. Express the area of the triangle as a monomial. 1. __________



For Questions 2 and 3, simplify each expression. Assume that no denominator is equal to zero.

2. $(-2x^4)^3(-4x^2y^6)^3$ 2. __________

3. $\frac{(3b^{-4}g^3)^{-2}}{(4b)^2}$ 3. __________

4. Arrange the terms in the polynomial $5xy^2 - 7x^4 + 3x^2y + 6$ so the powers of x are in ascending order. 4. __________

5. Find the degree of the polynomial $4hk + 2h - k^3$. 5. __________

6. Find $(4a - 2b + c) - (5c + a - 4b)$. 6. __________

7. Solve $x(x - 3) = x(x + 5) - 16$. 7. __________

For Questions 8–11, find each product.

8. $(p - 3)(p + 5)$ 8. __________

9. $(a + 2)(a^2 - 3a - 7)$ 9. __________

10. $(5t + s)^2$ 10. __________

11. $(3k - 1)(3k + 1)$ 11. __________

12. Find the GCF of $90m^2np^3$ and $150mn^3p^2$. 12. __________

For Questions 13–16, factor each polynomial.

13. $4p^2q - 16pq + 8pq^2$ 13. __________

14. $m^2 - 11m + 18$ 14. __________

15. $3a^2 - 13a + 12$ 15. __________

16. $27r^2 - 12s^2$ 16. __________

17. The area of a square is $4p^2 - 4p + 1$ square inches. What is the length of the side of the square? 17. __________

9 Unit 3 Test *(continued)*

For Questions 18–22, solve each equation.

18. $22y^2 = -11y$

18. ________

19. $h^2 - 13h + 36 = 0$

19. ________

20. $5a^2 - 33a = 14$

20. ________

21. $24n^3 = 6n$

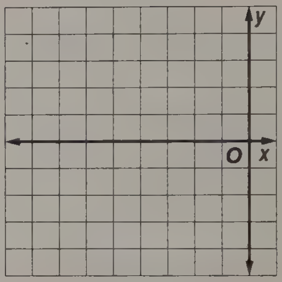
21. ________

22. $m^2 + 81 = -18m$

22. ________

23. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $y = x^2 + 8x + 12$. Then graph $y = x^2 + 8x + 12$.

23. ________



24. Solve $t^2 - 2t + 1 = 49$ by taking the square root of each side.

24. ________

25. State the value of the discriminant for $3x^2 + 2x + 1 = 0$.

25. ________

26. Solve $6 = 3t^2 + 2t$ by using the Quadratic Formula. Round to the nearest tenth if necessary.

26. ________

27. Determine whether the data in the table display exponential behavior. Explain why or why not.

x	5	10	15	20
y	0	2	4	6

27. ________

28. The population of Las Vegas, Nevada has been increasing at an annual rate of 7.0%. If the population of Las Vegas was 404,288 in the year 1999, predict its population in 2010.

28. ________

29. A new motor home costs $89,000. It is expected to depreciate 9% each year. Find the value of the motor home in 4 years.

29. ________

30. Find the total amount of an investment if $1200 is invested at an interest rate of 3.5% compounded quarterly for 7 years.

30. ________

NAME ______________ DATE ________ PERIOD ____

9 Anticipation Guide

Quadratic and Exponential Functions

Step 1 *Before you begin Chapter 9*

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

STEP 1 A, D, or NS	Statement	STEP 2 A or D
	1. The graph of a quadratic function is a parabola.	A
	2. The graph of $4x^2 - 2x + 7$ will be a parabola opening downward since the coefficient of x^2 is positive.	D
	3. A quadratic function's axis of symmetry is either the x-axis or the y-axis.	D
	4. The graph of a quadratic function opening upward has no maximum value.	A
	5. The x-intercepts of the graph of a quadratic function are the solutions to the related quadratic equation.	A
	6. All quadratic equations have two real solutions.	D
	7. Any quadratic expression can be written as a perfect square by a method called *completing the square.*	A
	8. The quadratic formula can only be used to solve quadratic equations that cannot be solved by factoring or graphing.	D
	9. A function containing powers is called an exponential function.	D
	10. Receiving compound interest on a bank account is one example of exponential growth.	A

Step 2 *After you complete Chapter 9*

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

NAME ______________ DATE ________ PERIOD ____

9-1 Lesson Reading Guide

Graphing Quadratic Functions

Get Ready for the Lesson

Read the introduction to Lesson 9-1 in your textbook.

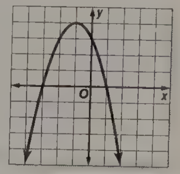
According to the graph, at what height does the rocket explode and in how many seconds after being launched? **It explodes at a height of 80 meters 4 seconds after being launched.**

Read the Lesson

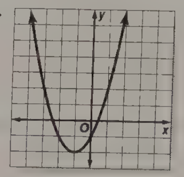
1. The standard form for a **quadratic** function is $y = ax^2 + bx + c$. For the function $y = 2x^2 - 5x + 3$, the value of a is **2**, the value of b is **−5**, and the value of c is **3**.

2. The graphs of two quadratic functions are shown below. Complete each statement about the graphs.

A.



B.



a. Each graph is a curve called a **parabola**.

b. The highest point of graph A is located at **(−1, 4)**. This point is the **maximum** (maximum/minimum) point of the graph.

c. The lowest point of graph B is located at **(−1, −2)**. This point is the **minimum** (maximum/minimum) point of the graph.

3. The maximum or minimum point of a parabola is called the **vertex** of the parabola.

4. If you fold a parabola along a line to get two halves that match exactly, the line where you fold the parabola is the **axis of symmetry** of the parabola. This line goes through the **vertex** of the parabola.

5. For a quadratic function $y = ax^2 + bx + c$, the parabola opens upward if a **>** 0. It opens downward if a **<** 0.

Remember What You Learned

6. Look up the word *vertex* in a dictionary. You will find that it comes from the Latin word *vertere*, which means to turn. How can you use the idea of "to turn" to remember what the vertex of a parabola is? **Sample answer: The vertex of a parabola is the point at which the parabola turns upward or downward.**

NAME ______________________ DATE ____________ PERIOD ____

9-1 Study Guide and Intervention

Graphing Quadratic Functions

Graph Quadratic Functions

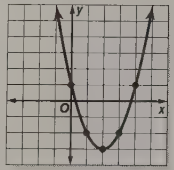
Quadratic Function	a function described by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$	**Example:** $y = 2x^2 + 3x + 8$

The parent graph of the family of quadratic fuctions is $y = x^2$. Graphs of quadratic functions have a general shape called a **parabola**. A parabola opens upward and has a **minimum point** when the value of a is positive, and a parabola opens downward and has a **maximum point** when the value of a is negative.

Example 1

a. Use a table of values to graph $y = x^2 - 4x + 1$.

x	y
-1	6
0	1
1	2
2	-3
3	-2
4	1



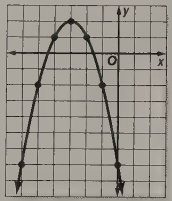
Graph the ordered pairs in the table and connect them with a smooth curve.

b. What is the domain and range of this function? D: $\{x \mid x$ is a real number.$\}$ R: $\{y \mid y \geq -3\}$

Example 2

a. Use a table of values to graph $y = -x^2 - 6x - 7$.

x	y
-6	-7
-5	-2
4	1
-3	2
-2	1
-1	-2
0	7



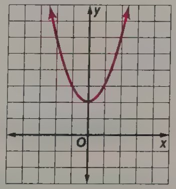
Graph the ordered pairs in the table and connect them with a smooth curve.

b. What is the domain and range of this function? D: $\{x \mid x$ is a real number.$\}$ R: $\{y \mid y \leq 2\}$

Exercises

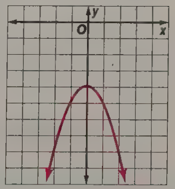
Use a table of values to graph each function. Determine the domain and range.

1. $y = x^2 + 2$



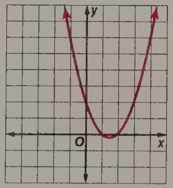
D: $\{x \mid x$ is a real number.$\}$ R: $\{y \mid y \geq 2\}$

2. $y = -x^2 - 4$



D: $\{x \mid x$ is a real number.$\}$ R: $\{y \mid y \leq -4\}$

3. $y = x^2 - 3x + 2$



D: $\{x \mid x$ is a real number.$\}$ R: $\{y \mid y \geq -\frac{1}{4}\}$

NAME ______________________ DATE ____________ PERIOD ____

9-1 Study Guide and Intervention *(continued)*

Graphing Quadratic Functions

Symmetry and Vertices Parabolas have a geometric property called **symmetry**. That is, if the figure is folded in half, each half will match the other half exactly. The vertical line containing the fold line is called the **axis of symmetry**.

Axis of Symmetry	For the parabola $y = ax^2 + bx + c$, where $a \neq 0$, the line $x = -\frac{b}{2a}$ is the axis of symmetry.	**Example:** The axis of symmetry of $y = x^2 + 2x + 5$ is the line $x = -1$.

The axis of symmetry contains the minimum or maximum point of the parabola, the **vertex**.

Example **Consider the graph of $y = 2x^2 + 4x + 1$.**

a. Write the equation of the axis of symmetry.

In $y = 2x^2 + 4x + 1$, $a = 2$ and $b = 4$. Substitute these values into the equation of the axis of symmetry.

$$x = -\frac{b}{2a}$$

$$x = -\frac{4}{2(2)} = -1$$

The axis of symmetry is $x = -1$.

c. Identify the vertex as a maximum or a minimum.

Since the coefficient of the x^2-term is positive, the parabola opens upward, and the vertex is a minimum point.

b. Find the coordinates of the vertex.

Since the equation of the axis of symmetry is $x = -1$ and the vertex lies on the axis, the x-coordinate of the vertex is -1.

$y = 2x^2 + 4x + 1$ Original equation

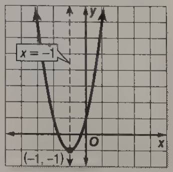
$y = 2(-1)^2 + 4(-1) + 1$ Substitute.

$y = 2(1) - 4 + 1$ Simplify.

$y = -1$

The vertex is at $(-1, -1)$.

d. Graph the function.

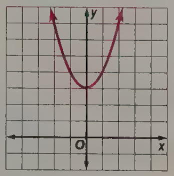


Exercises

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or a minimum. Then graph the function.

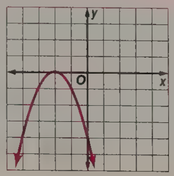
1. $y = x^2 + 3$

$x = 0$; $(0, 3)$; min



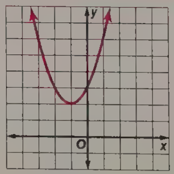
2. $y = -x^2 - 4x - 4$

$x = -2$; $(-2, 0)$; max



3. $y = x^2 + 2x + 3$

$x = -1$; $(-1, 2)$; min



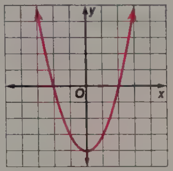
NAME ______________________ DATE __________ PERIOD _____

9-1 Skills Practice

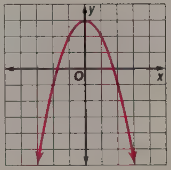
Graphing Quadratic Functions

Use a table of values to graph each function.

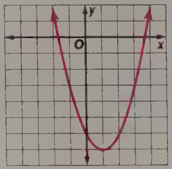
1. $y = x^2 - 4$



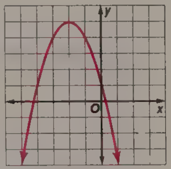
2. $y = -x^2 + 3$



3. $y = x^2 - 2x - 6$

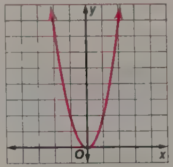


4. $y = -x^2 - 4x + 1$

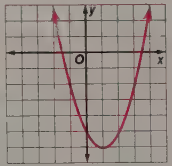


Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

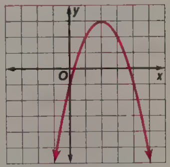
5. $y = 2x^2$
$x = 0$; (0, 0); min



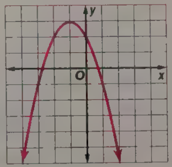
6. $y = x^2 - 2x - 5$
$x = 1$; (1, −6); min



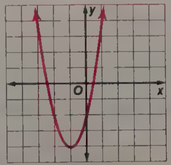
7. $y = -x^2 + 4x - 1$
$x = 2$; (2, 3); max



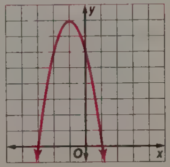
8. $y = -x^2 - 2x + 2$
$x = -1$; (−1, 3); max



9. $y = 2x^2 + 4x - 2$
$x = -1$; (−1, −4); min



10. $y = -2x^2 - 4x + 6$
$x = -1$; (−1, 8); max



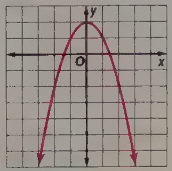
NAME ______________________ DATE __________ PERIOD _____

9-1 Practice

Graphing Quadratic Functions

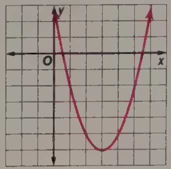
Use a table of values to graph each function. Determine the domain and range.

1. $y = -x^2 + 2$



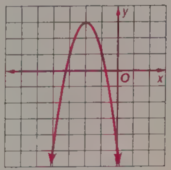
D: $\{x \mid x$ is a real number.$\}$ R: $\{y \mid y \leq 2\}$

2. $y = x^2 - 6x + 3$



D: $\{x \mid x$ is a real number.$\}$ R: $\{y \mid y \geq -6\}$

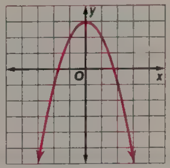
3. $y = -2x^2 - 8x - 5$



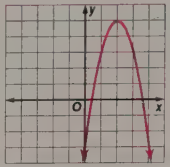
D: $\{x \mid x$ is a real number.$\}$ R: $\{y \mid y \leq 3\}$

Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of each function. Identify the vertex as a maximum or minimum. Then graph the function.

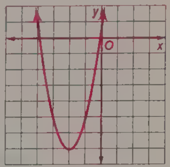
4. $y = -x^2 + 3$
$x = 0$; (0, 3); max



5. $y = -2x^2 + 8x - 3$
$x = 2$; (2, 5); max



6. $y = 2x^2 + 8x + 1$
$x = -2$; (−2, −7); min



PHYSICS For Exercises 7–9, use the following information.

Miranda throws a set of keys up to her brother, who is standing on a balcony 38 feet above the ground. The equation $h = -16t^2 + 40t + 5$ gives the height h of the keys after t seconds.

7. How long does it take the keys to reach their highest point? 1.25 s

8. How high do the keys reach? 30 ft

9. Will her brother be able to catch the keys? Explain. No, the keys will be 8 ft short of their target.

10. What is a reasonable domain and range for this problem? D: $\{x \mid x \geq 0\}$
R: $\{y \mid 0 \leq y \leq 30\}$

BASEBALL For Exercises 11–13, use the following information.

A player hits a baseball into the outfield. The equation $h = -0.005x^2 + x + 3$ gives the path of the ball, where h is the height and x is the horizontal distance the ball travels.

11. What is the equation of the axis of symmetry? $x = 100$

12. What is the maximum height reached by the baseball? 53 ft

13. An outfielder catches the ball three feet above the ground. How far has the ball traveled horizontally when the outfielder catches it? 200 ft

NAME ______________________ DATE __________ PERIOD _____

9-1 Word Problem Practice

Graphing Quadratic Functions

1. **OLYMPICS** Olympics were held in 1896 and have been held every four years (except 1916, 1940, and 1944). The winning height y in men's pole vault at any number Olympiad x can be approximated by the equation $y = 0.37x^2 + 4.3x + 126$. Complete the table to estimate the pole vault heights in each of the Olympic Games. Round your answers to the nearest tenth.

Year	Olympiad (x)	Height (y inches)
1896	1	130.7
1900	2	136.1
1924	7	174.2
1936	10	206.0
1964	15	273.8
2008	26	487.9

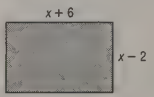
Source: www.nsa.gov/teachers/hs/alg01.pdf

2. **PHYSICS** Mrs. Capwell's physics class investigates what happens when a ball is given an initial push, rolls up, and then back down an inclined plane. The class finds that $y = -x^2 + 6x$ accurately predicts the ball's position y after rolling x seconds. On the graph of the equation, what would be the y value when $x = 4$?
8

3. **ARCHITECTURE** A hotel's main entrance is in the shape of a parabolic arch. The equation $y = -x^2 + 10x$ models the arch height y for any distance x from one side of the arch. Use a graph to determine its maximum height. 25 ft

4. **SOFTBALL** Olympic softball gold medalist Michele Smith pitches a curveball with a speed of 64 feet per second. If she throws the ball straight upward at this speed, the ball's height h (in feet) after t seconds is given by $h = -16t^2 + 64t$. Find the coordinates of the vertex of the graph of the ball's height and interpret its meaning.
(2, 64); After 2 seconds, the ball reaches its highest point, 64 ft above the ground.

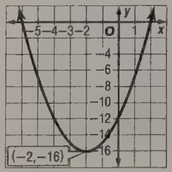
GEOMETRY For Exercises 5–7, use the following information.
Teddy is building the rectangular deck shown below.



5. Write the equation representing the area of the deck. $y = (x - 2)(x + 6)$ or $y = x^2 + 4x - 12$

6. What is the equation of the axis of symmetry? $x = -2$

7. Graph the equation and label its vertex.



Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

NAME ______________________ DATE __________ PERIOD _____

9-1 Enrichment

Translating Quadratic Graphs

When a figure is moved to a new position without undergoing any rotation, then the figure is said to have been **translated** to that position.

The graph of a quadratic equation in the form $y = (x - b)^2 + c$ is a translation of the graph of $y = x^2$.

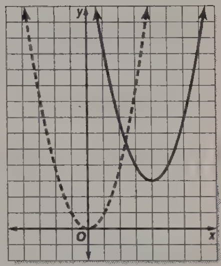
Start with $y = x^2$.

Slide to the right 4 units.

$y = (x - 4)^2$

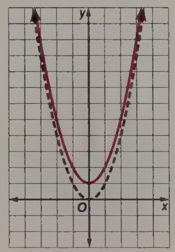
Then slide up 3 units.

$y = (x - 4)^2 + 3$

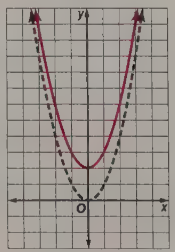


These equations have the form $y = x^2 + c$. Graph each equation.

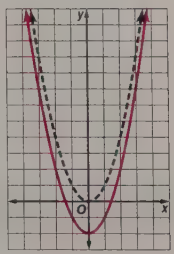
1. $y = x^2 + 1$



2. $y = x^2 + 2$

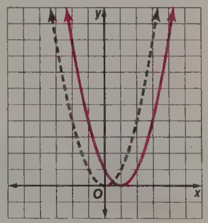


3. $y = x^2 - 2$

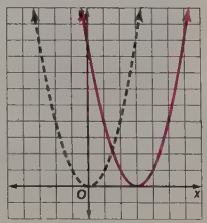


These equations have the form $y = (x - b)^2$. Graph each equation.

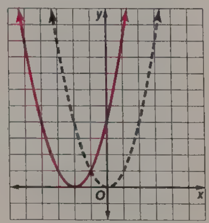
4. $y = (x - 1)^2$



5. $y = (x - 3)^2$



6. $y = (x + 2)^2$



Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

NAME ______________________ DATE ____________ PERIOD _____

9-2 Lesson Reading Guide

Solving Quadratic Equations by Graphing

Get Ready for the Lesson

Read the introduction to Lesson 9-2 in your textbook.

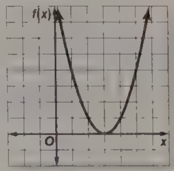
If one of the x-intercepts represents the location where the ball will hit the ground, what does the other x-intercept represent? The location where the ball is hit.

Read the Lesson

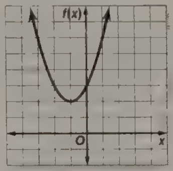
1. The x-intercepts of the graph of a quadratic function are the x-coordinates of the points where the graph of the function intersects the x-axis. At those points, the y-coordinates are equal to ___0___. This explains why the x-intercepts are called ___zeros___ of the quadratic function.

2. The graphs of three functions are shown below. Use the graphs to provide the requested information about the related quadratic equations.

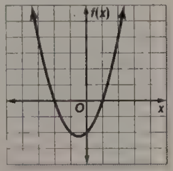
A. $f(x) = x^2 - 6x + 9$



B. $f(x) = x^2 + 2x + 3$



C. $f(x) = x^2 + x - 2$



a. For Graph A, the related quadratic equation is ___$x^2 - 6x + 9 = 0$___.

How many real solutions are there? one

Name any solutions. 3

b. For Graph B, the related quadratic equation is ___$x^2 + 2x + 3 = 0$___.

How many real solutions are there? none

Name any solutions. none

c. For Graph C, the related quadratic equation is ___$x^2 + x - 2 = 0$___.

How many real solutions are there? two

Name any solutions. −2 and 1

Remember What You Learned

3. Describe how you can remember that the word *zero* is used when you are talking about functions, but the word *root* is used when you are talking about equations. Sample answer: Some functions can have a value of zero. Equations can be true or false, but they do not have a number value.

NAME ______________________ DATE ____________ PERIOD _____

9-2 Study Guide and Intervention

Solving Quadratic Equations by Graphing

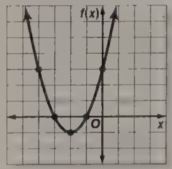
Solve by Graphing

Quadratic Equation	an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$

The solutions of a quadratic equation are called the **roots** of the equation. The roots of a quadratic equation can be found by graphing the related quadratic function $f(x) = ax^2 + bx + c$ and finding the x-intercepts or **zeros** of the function.

Example 1 **Solve $x^2 + 4x + 3 = 0$ by graphing.**

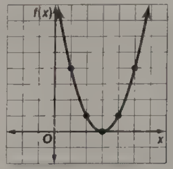
Graph the related function $f(x) = x^2 + 4x + 3$. The equation of the axis of symmetry is $x = -\frac{4}{2(1)}$ or −2. The vertex is at (−2, −1). Graph the vertex and several other points on either side of the axis of symmetry.



To solve $x^2 + 4x + 3 = 0$, you need to know where the value of $f(x) = 0$. This occurs at the x-intercepts, −3 and −1. The solutions are −3 and −1.

Example 2 **Solve $x^2 - 6x + 9 = 0$ by graphing.**

Graph the related function $f(x) = x^2 - 6x + 9$. The equation of the axis of symmetry is $x = \frac{6}{2(1)}$ or 3. The vertex is at (3, 0). Graph the vertex and several other points on either side of the axis of symmetry.

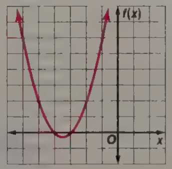


To solve $x^2 - 6x + 9 = 0$, you need to know where the value of $f(x) = 0$. The vertex of the parabola is the x-intercept. Thus, the only solution is 3.

Exercises

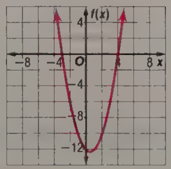
Solve each equation by graphing.

1. $x^2 + 7x + 12 = 0$



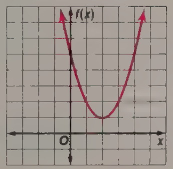
−3; −4

2. $x^2 - x - 12 = 0$



4, −3

3. $x^2 - 4x + 5 = 0$



no real roots

NAME ______________________ DATE __________ PERIOD _____

9-2 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Graphing

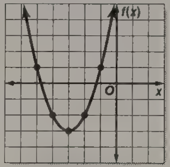
Estimate Solutions The roots of a quadratic equation may not be integers. If exact roots cannot be found, they can be estimated by finding the consecutive integers between which the roots lie.

Example **Solve $x^2 + 6x + 6 = 0$ by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.**

Graph the related function $f(x) = x^2 + 6x + 6$.

x	$f(x)$
−5	1
−4	−2
−3	−3
−2	−2
−1	1

Notice that the value of the function changes from negative to positive between the x-values of −5 and −4 and between −2 and −1.

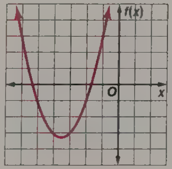


The x-intercepts of the graph are between −5 and −4 and between −2 and −1. So one root is between −5 and −4, and the other root is between −2 and −1.

Exercises

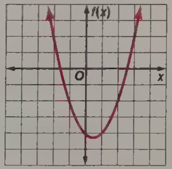
Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

1. $x^2 + 7x + 9 = 0$



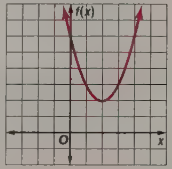
$-6 < x < -5$, $-2 < x < -1$

2. $x^2 - x - 4 = 0$



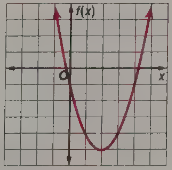
$-2 < x < -1$, $2 < x < 3$

3. $x^2 - 4x + 6 = 0$



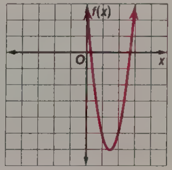
no real roots

4. $x^2 - 4x - 1 = 0$



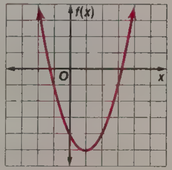
$-1 < x < 0$, $4 < x < 5$

5. $4x^2 - 12x + 3 = 0$



$0 < x < 1$, $2 < x < 3$

6. $x^2 - 2x - 4 = 0$



$-2 < x < -1$, $3 < x < 4$

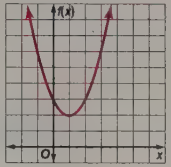
NAME ______________________ DATE __________ PERIOD _____

9-2 Skills Practice

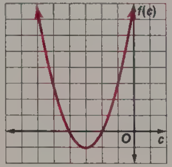
Solving Quadratic Equations by Graphing

Solve each equation by graphing.

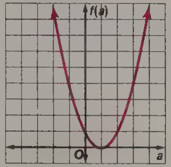
1. $x^2 - 2x + 3 = 0$ $\varnothing$



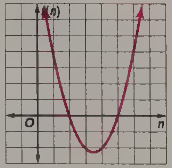
2. $c^2 + 6c + 8 = 0$ −4, −2



3. $a^2 - 2a = -1$ 1

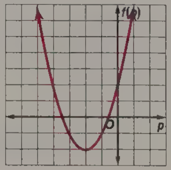


4. $n^2 - 7n = -10$ 2, 5



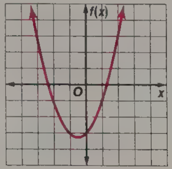
Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

5. $p^2 + 4p + 2 = 0$



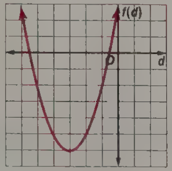
$-4 < p < -3$, $-1 < p < 0$

6. $x^2 + x - 3 = 0$



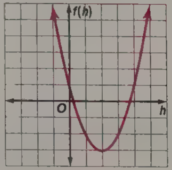
$-3 < x < -2$, $1 < x < 2$

7. $d^2 + 6d = -3$



$-6 < d < -5$, $-1 < d < 0$

8. $h^2 + 1 = 4h$



$0 < h < 1$, $3 < h < 4$

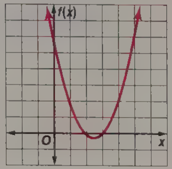
NAME ______________________ DATE__________ PERIOD _____

9-2 Practice

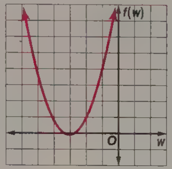
Solving Quadratic Equations by Graphing

Solve each equation by graphing.

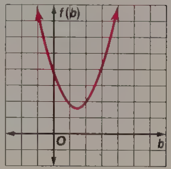
1. $x^2 - 5x + 6 = 0$ **2, 3**



2. $w^2 + 6w + 9 = 0$ **−3**

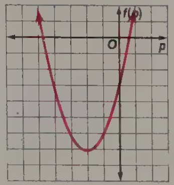


3. $b^2 - 3b + 4 = 0$ **∅**



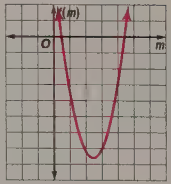
Solve each equation by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

4. $p^2 + 4p = 3$



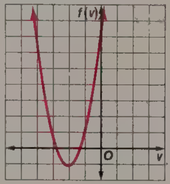
$-5 < p < -4$, $0 < p < 1$

5. $2m^2 + 5 = 10m$



$0 < m < 1$, $4 < m < 5$

6. $2v^2 + 8v = -7$

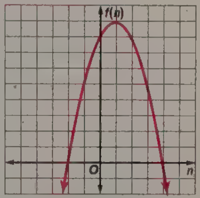


$-3 < v < -2$, $-2 < v < -1$

NUMBER THEORY For Exercises 7 and 8, use the following information.

Two numbers have a sum of 2 and a product of −8. The quadratic equation $-n^2 + 2n + 8 = 0$ can be used to determine the two numbers.

7. Graph the related function $f(n) = -n^2 + 2n + 8$ and determine its x-intercepts. **−2, 4**

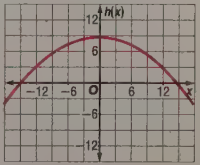


8. What are the two numbers? **−2 and 4**

DESIGN For Exercises 9 and 10, use the following information.

A footbridge is suspended from a parabolic support. The function $h(x) = -\frac{1}{25}x^2 + 9$ represents the height in feet of the support above the walkway, where $x = 0$ represents the midpoint of the bridge.

9. Graph the function and determine its x-intercepts. **−15, 15**



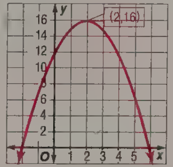
10. What is the length of the walkway between the two supports? **30 ft**

NAME ______________________ DATE__________ PERIOD _____

9-2 Word Problem Practice

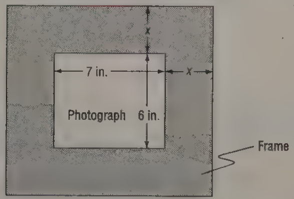
Solving Quadratic Equations by Graphing

1. FARMING In order for Ray to decide how much fertilizer to apply to his corn crop this year, he reviews records from previous years. He finds that his crop yield y depends on the amount of fertilizer he applies to his fields x according to the equation $y = -x^2 + 4x + 12$. Graph the function, and find the point at which Ray gets the highest yield possible.



2. LIGHT Ayzha and Jeremy hold a flashlight so that the light falls on a piece of graph paper in the shape of a parabola. Ayzha and Jeremy sketch the shape of the parabola and find that the equation $y = x^2 - 3x - 10$ matches the shape of the light beam. Determine the roots of the function. **−2 and 5**

3. FRAMING A rectangular photograph is 7 inches long and 6 inches wide. The photograph is framed using material x inches wide. If the area of the frame and photograph combined is 156 square inches, what is the width of the framing material? **3 in.**



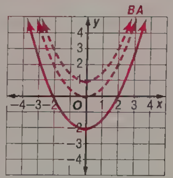
4. WRAPPING PAPER Can a rectangular piece of wrapping paper with an area of 81 square inches have a perimeter of 60 inches? (*Hint:* Let length = 30 − *w*.) Explain. **Solving the equation $(30 - w)w = 81$ gives $w = 3$ or 27. A 3 in. by 27 in. sheet of paper would work.**

ENGINEERING For Exercises 5–7, use the following information.

The shape of a satellite dish is often parabolic because of the reflective qualities of parabolas. Suppose a particular satellite dish is modeled by the following equation.

$$0.5x^2 = 2 + y$$

5. Approximate the solution by graphing. **−2 and 2**



6. On the coordinate plane above, translate the parabola so that there is a double root. Label this curve A. **See students' work.**

7. Translate the parabola so that there are no roots. Label this curve B. **See students' work.**

NAME ______________________________ DATE ____________ PERIOD _____

9-2 Enrichment

Rational Exponents

You have developed the following properties of powers when a is a positive real number and m and n are integers.

$a^m \cdot a^n = a^{m+n}$ $\quad (ab)^m = a^m b^m$ $\quad a^0 = 1$

$(a^m)^n = a^{mn}$ $\quad \frac{a^m}{a^n} = a^{m-n}$ $\quad a^{-m} = \frac{1}{a^m}$

Exponents need not be restricted to integers. We can define rational exponents so that operations involving them will be governed by the properties for integer exponents.

$\left(a^{\frac{1}{2}}\right)^2 = a^{\frac{1}{2} \cdot 2} = a$	$\left(a^{\frac{1}{3}}\right)^3 = a^{\frac{1}{3} \cdot 3}$	$\left(a^{\frac{1}{n}}\right)^n = a^{\frac{1}{n} \cdot n} = a$
$a^{\frac{1}{2}}$ squared is a.	$a^{\frac{1}{3}}$ cubed is a.	$a^{\frac{1}{n}}$ to the n power is a.
$a^{\frac{1}{2}}$ is a square root of a.	$a^{\frac{1}{3}}$ is a cube root of a.	$a^{\frac{1}{n}}$ is an nth root of a.
$a^{\frac{1}{2}} = \sqrt{a}$	$a^{\frac{1}{3}} = \sqrt[3]{a}$	$a^{\frac{1}{n}} = \sqrt[n]{a}$

Now let us investigate the meaning of $a^{\frac{m}{n}}$.

$a^{\frac{m}{n}} = a^{m \cdot \frac{1}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$ $\qquad a^{\frac{m}{n}} = a^{\frac{1}{n} \cdot m} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$

Therefore, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m$.

Example 1 **Write $\sqrt[4]{a^3}$ in exponential form.**

$\sqrt[4]{a^3} = a^{\frac{3}{4}}$

Example 2 **Write $a^{\frac{2}{5}}$ in radical form.**

$a^{\frac{2}{5}} = \sqrt[5]{a^2}$

Example 3 **Find $\frac{a^{\frac{2}{3}}}{a^{\frac{1}{2}}}$.**

$\frac{a^{\frac{2}{3}}}{a^{\frac{1}{2}}} = a^{\frac{2}{3} - \frac{1}{2}} = a^{\frac{4}{6} - \frac{3}{6}} = a^{\frac{1}{6}}$ or $\sqrt[6]{a}$

Write each expression in radical form.

1. $b^{\frac{2}{3}}$ $\sqrt[3]{b^2}$ $\quad$ **2.** $3c^{\frac{1}{2}}$ $3\sqrt{c}$ $\quad$ **3.** $(3c)^{\frac{1}{2}}$ $\sqrt{3c}$

Write each expression in exponential form.

4. $\sqrt[3]{b^4}$ $b^{\frac{4}{3}}$ $\quad$ **5.** $\sqrt{4a^3}$ $(4a^3)^{\frac{1}{2}} = 2a^{\frac{3}{2}}$ $\quad$ **6.** $2 \cdot \sqrt[3]{b^2}$ $2b^{\frac{2}{3}}$

Perform the operation indicated. Answers should show positive exponents only.

7. $\left(a^3 b^{\frac{1}{4}}\right)^2$ $a^6 b^{\frac{1}{2}}$ $\quad$ **8.** $\frac{-8a^{\frac{3}{4}}}{2a^{\frac{1}{2}}}$ $-4a^{\frac{1}{4}}$ $\quad$ **9.** $\left(\frac{b^{\frac{1}{2}}}{b^{-\frac{2}{3}}}\right)^3$ $b^{\frac{7}{2}}$

10. $\sqrt{a^3} \cdot \sqrt{a}$ a^2 $\quad$ **11.** $\left(a^2 b^{-\frac{1}{3}}\right)^{-\frac{1}{2}}$ $\frac{b^{\frac{1}{6}}}{a}$ $\quad$ **12.** $-2a^{\frac{1}{3}}b^0\left(5a^{\frac{1}{2}}b^{-\frac{2}{3}}\right)$ $\frac{-10a^{\frac{5}{6}}}{b^{\frac{2}{3}}}$

NAME ______________________________ DATE ____________ PERIOD _____

9-3 Lesson Reading Guide

Solving Quadratic Equations by Completing the Square

Get Ready for the Lesson

Read the introduction to Lesson 9-3 in your textbook.

To solve the problem, how many "units" would Al-Khwarizmi have added to each side of the equation? **16**

Read the Lesson

1. Draw a line under each quadratic equation that you could solve by taking the square root of each side.

$\underline{x^2 + 6x + 9 = 100}$	$x^2 - 14x + 40 = 25$	$\underline{x^2 - 16x + 64 = 26}$
$x^2 - 20x + 80 = 16$	$x^2 + 10x + 36 = 49$	$\underline{x^2 - 12x + 36 = 6}$

2. How can you tell whether it is possible to solve a quadratic equation by taking the square root of each side?

Sample answer: Check to be sure that all terms that contain the variable are on the same side of the equation. Next, check whether the quadratic expression is a perfect square. Finally, check that the number on the side that contains no variables is greater than or equal to 0.

3. Explain how to find what number is needed for the ■ in order to make $x^2 - 20x +$ ■ a perfect square.

Find one half of −20, which is −10, and square it. The result, which is 100, is the required number.

4. To solve $3x^2 - 6x = 54$ by completing the square, why does it help first to divide both sides by 3?

Sample answer: If you divide both sides by 3, the coefficient of x^2 will be 1. It is then easy to use the coefficient of x to decide what number you need to add to both sides to make the left side a perfect square trinomial.

Remember What You Learned

5. The method of completing the square might be easier to remember if you can connect it to what you know about perfect square trinomials. How is completing the square related to the method you use to determine whether a trinomial is a perfect square trinomial?

Sample answer: If the first and last terms of a trinomial are perfect squares, you *multiply* the product of their *square roots by 2* to get the middle term. In completing the square, you check that the coefficient of x^2 is 1. Then you divide the coefficient of the x term by 2 and square the result to get the third term.

NAME ______________________ DATE __________ PERIOD ____

9-3 Study Guide and Intervention

Solving Quadratic Equations by Completing the Square

Find the Square Root An equation such as $x^2 - 4x + 4 = 5$ can be solved by taking the square root of each side.

Example 1 **Solve $x^2 - 2x + 1 = 9$. Round to the nearest tenth if necessary.**

$$\begin{aligned} x^2 - 2x + 1 &= 9 \\ (x-1)^2 &= 9 \\ \sqrt{(x-1)^2} &= \sqrt{9} \\ |x-1| &= \sqrt{9} \\ x - 1 &= \pm 3 \\ x - 1 + 1 &= \pm 3 + 1 \\ x &= 1 \pm 3 \end{aligned}$$

$x = 1 + 3$ or $x = 1 - 3$
$= 4$ $= -2$

The solution set is $\{-2, 4\}$.

Example 2 **Solve $x^2 - 4x + 4 = 5$. Round to the nearest tenth if necessary.**

$$\begin{aligned} x^2 - 4x + 4 &= 5 \\ (x-2)^2 &= 5 \\ \sqrt{(x-2)^2} &= \sqrt{5} \\ |x-2| &= \sqrt{5} \\ x - 2 &= \pm\sqrt{5} \\ x - 2 + 2 &= \pm\sqrt{5} + 2 \\ x &= 2 \pm \sqrt{5} \end{aligned}$$

Use a calculator to evaluate each value of x.

$x = 2 + \sqrt{5}$ or $x = 2 - \sqrt{5}$
≈ 4.2 ≈ -0.2

The solution set is $\{-0.2, 4.2\}$.

Exercises

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $x^2 + 4x + 4 = 9$ **−5, 1**
2. $m^2 + 12m + 36 = 1$ **−7, −5**
3. $r^2 - 6r + 9 = 16$ **−1, 7**
4. $x^2 - 2x + 1 = 25$ **−4, 6**
5. $x^2 - 8x + 16 = 5$ **1.8, 6.2**
6. $x^2 - 10x + 25 = 8$ **2.2, 7.8**
7. $c^2 - 4c + 4 = 7$ **−0.6, 4.6**
8. $p^2 + 16p + 64 = 3$ **−9.7, −6.3**
9. $x^2 + 8x + 16 = 9$ **−7, −1**
10. $x^2 + 6x + 9 = 4$ **−5, −1**
11. $a^2 + 8a + 16 = 10$ **−7.2, −0.8**
12. $y^2 - 12y + 36 = 5$ **3.8, 8.2**
13. $x^2 + 10x + 25 = 1$ **−6, −4**
14. $y^2 + 14y + 49 = 6$ **−9.4, −4.6**
15. $m^2 - 8m + 16 = 2$ **2.6, 5.4**
16. $x^2 + 12x + 36 = 10$ **−9.2, −2.8**
17. $a^2 - 14a + 49 = 3$ **5.3, 8.7**
18. $y^2 + 8y + 16 = 7$ **−6.6, −1.4**

NAME ______________________ DATE __________ PERIOD ____

9-3 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Completing the Square

Complete the Square Since few quadratic expressions are perfect square trinomials, the method of **completing the square** can be used to solve some quadratic equations. Use the following steps to complete the square for a quadratic expression of the form $ax^2 + bx$.

Step 1	Find $\frac{b}{2}$.
Step 2	Find $\left(\frac{b}{2}\right)^2$.
Step 3	Add $\left(\frac{b}{2}\right)^2$ to $ax^2 + bx$.

Example **Solve $x^2 + 6x + 3 = 10$ by completing the square.**

$x^2 + 6x + 3 = 10$	Original equation
$x^2 + 6x + 3 - 3 = 10 - 3$	Subtract 3 from each side.
$x^2 + 6x = 7$	Simplify.
$x^2 + 6x + 9 = 7 + 9$	Since $\left(\frac{6}{2}\right)^2 = 9$, add 9 to each side.
$(x + 3)^2 = 16$	Factor $x^2 + 6x + 9$.
$x + 3 = \pm 4$	Take the square root of each side.
$x = -3 \pm 4$	Simplify.

$x = -3 + 4$ or $x = -3 - 4$
$= 1$ $= -7$

The solution set is $\{-7, 1\}$.

Exercises

Solve each equation by completing the square. Round to the nearest tenth if necessary.

1. $t^2 - 4t + 3 = 0$ **1, 3**
2. $y^2 + 10y = -9$ **−1, −9**
3. $y^2 - 8y - 9 = 0$ **−1, 9**
4. $x^2 - 6x = 16$ **−2, 8**
5. $p^2 - 4p - 5 = 0$ **−1, 5**
6. $x^2 - 12x = 9$ **−0.7, 12.7**
7. $c^2 + 8c = 20$ **−10, 2**
8. $p^2 = 2p + 1$ **−0.4, 2.4**
9. $x^2 + 20x + 11 = -8$ **−19, −1**
10. $x^2 - 1 = 5x$ **−0.2, 5.2**
11. $a^2 = 22a + 23$ **−1, 23**
12. $m^2 - 8m = -7$ **1, 7**
13. $x^2 + 10x = 24$ **−12, 2**
14. $a^2 - 18a = 19$ **−1, 19**
15. $b^2 + 16b = -16$ **−14.9, −1.1**
16. $4x^2 = 24 + 4x$ **−2, 3**
17. $2m^2 + 4m + 2 = 8$ **−3, 1**
18. $4k^2 = 40k + 44$ **−1, 11**

NAME ______________________ DATE ____________ PERIOD _____

9-3 Skills Practice

Solving Quadratic Equations by Completing the Square

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $c^2 - 12c + 36 = 4$ **4, 8**

2. $w^2 - 10w + 25 = 16$ **1, 9**

3. $b^2 + 16b + 64 = 9$ **−11, −5**

4. $y^2 + 2y + 1 = 3$ **−2.7, 0.7**

5. $r^2 + 4r + 4 = 7$ **−4.6, 0.6**

6. $a^2 - 8a + 16 = 12$ **0.5, 7.5**

Find the value of *c* that makes each trinomial a perfect square.

7. $g^2 + 6g + c$ **9**

8. $y^2 + 4y + c$ **4**

9. $a^2 - 14a + c$ **49**

10. $n^2 - 2n + c$ **1**

11. $s^2 - 18s + c$ **81**

12. $p^2 + 20p + c$ **100**

Solve each equation by completing the square. Round to the nearest tenth if necessary.

13. $x^2 + 4x - 12 = 0$ **2, −6**

14. $v^2 - 8v + 15 = 0$ **3, 5**

15. $q^2 + 6q = 7$ **−7, 1**

16. $r^2 - 2r = 15$ **−3, 5**

17. $m^2 - 14m + 30 = 6$ **2, 12**

18. $b^2 + 12b + 21 = 10$ **−11, −1**

19. $z^2 - 4z + 1 = 0$ **0.3, 3.7**

20. $y^2 - 6y + 4 = 0$ **0.8, 5.2**

21. $r^2 - 8r + 10 = 0$ **1.6, 6.4**

22. $p^2 - 2p = 5$ **−1.4, 3.4**

23. $2a^2 + 20a = -2$ **−9.9, −0.1**

24. $0.5g^2 + 8g = -7$ **−15.1, −0.9**

NAME ______________________ DATE ____________ PERIOD _____

9-3 Practice

Solving Quadratic Equations by Completing the Square

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $b^2 - 14b + 49 = 64$ **−1, 15**

2. $s^2 + 16s + 64 = 100$ **−18, 2**

3. $h^2 - 8h + 16 = 15$ **0.1, 7.9**

4. $a^2 + 6a + 9 = 27$ **−8.2, 2.2**

5. $p^2 - 20p + 100 = 28$ **4.7, 15.3**

6. $u^2 + 10u + 25 = 90$ **−14.5, 4.5**

Find the value of *c* that makes each trinomial a perfect square.

7. $t^2 - 24t + c$ **144**

8. $b^2 + 28b + c$ **196**

9. $y^2 + 40y + c$ **400**

10. $m^2 + 3m + c$ $\frac{9}{4}$

11. $g^2 - 9g + c$ $\frac{81}{4}$

12. $v^2 - v + c$ $\frac{1}{4}$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

13. $w^2 - 14w + 24 = 0$ **2, 12**

14. $p^2 + 12p = 13$ **−13, 1**

15. $s^2 - 30s + 56 = -25$ **3, 27**

16. $v^2 + 8v + 9 = 0$ **−6.6, −1.4**

17. $t^2 - 10t + 6 = -7$ **1.5, 8.5**

18. $n^2 + 18n + 50 = 9$ **−15.3, −2.7**

19. $3u^2 + 15u - 3 = 0$ **−5.2, 0.2**

20. $4c^2 - 72 = 24c$ **−2.2, 8.2**

21. $0.9a^2 + 5.4a - 4 = 0$ $-6\frac{2}{3}, \frac{2}{3}$

22. $0.4h^2 + 0.8h = 0.2$ **−2.2, 0.2**

23. $\frac{1}{2}x^2 - \frac{1}{2}x - 10 = 0$ **−4, 5**

24. $\frac{1}{4}x^2 + \frac{3}{2}x - 2 = 0$ **−7.1, 1.1**

BUSINESS For Exercises 25 and 26, use the following information.

Jaime owns a business making decorative boxes to store jewelry, mementos, and other valuables. The function $y = x^2 + 50x + 1800$ models the profit *y* that Jaime has made in month *x* for the first two years of his business.

25. Write an equation representing the month in which Jaime's profit is $2400.
$x^2 + 50x + 1800 = 2400$

26. Use completing the square to find out in which month Jaime's profit is $2400.
the tenth month

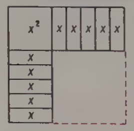
27. **PHYSICS** From a height of 256 feet above a lake on a cliff, Mikaela throws a rock out over the lake. The height *H* of the rock *t* seconds after Mikaela throws it is represented by the equation $H = -16t^2 + 32t + 256$. To the nearest tenth of a second, how long does it take the rock to reach the lake below? (*Hint:* Replace *H* with 0.) **5.1 s**

NAME ______________________ DATE __________ PERIOD _____

9-3 Word Problem Practice

Solving Quadratic Equations by Completing the Square

1. **INTERIOR DESIGN** Modular carpeting is installed in small pieces rather than as a large roll so that only a few pieces need to be replaced if a small area is damaged. Suppose the room shown in the diagram below is being fitted with modular carpeting. Complete the square to determine the number of 1 ft by 1 ft squares of carpeting needed to finish the room. Fill in the missing terms in the corresponding equation below. 25; 5



$x^2 + 10x +$ _____ $= (x +$ _____$)^2$

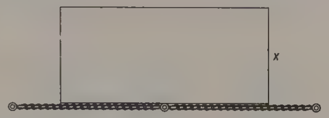
2. **FALLING OBJECTS** Keisha throws a rock down an old well. The distance d (in feet) the rock falls after t seconds can be represented by the equation $d = 16t^2 + 64t$. If the water in the well is 80 feet below ground, how many seconds will it take for the rock to hit the water?
   1 second

3. **MARS** On Mars, the gravity acting on an object is less than that on Earth. On Earth, a golf ball hit with an initial upward velocity of 26 meters per second will reach a maximum height of about 34.5 meters. The height h of an object on Mars that leaves the ground with an initial velocity of 26 meters per second is given by the equation $h = -1.9t^2 + 26t$. Find the maximum height if the same golf ball is hit on Mars. Round your answer to the nearest tenth. 88.9 m

4. **FROGS** A frog sitting on a stump 3 feet high hops off and lands on the ground. During its leap, its height h (in feet) is given by $h = -0.5d^2 + 2d + 3$, where d is the distance from the base of the stump. How far is the frog from the base of the stump when it landed on the ground?
   $2 + \sqrt{10}$ or about 5.16 ft

GARDENING For exercises 5–7, use the following information.

Peg is planning a rectangular vegetable garden using 250 feet of fencing material. She only needs to fence three sides of the garden since one side borders an existing fence.



5. Let x = the width of the rectangle. Write an expression to represent the area of the garden if she uses all the fencing material. $x(250 - 2x)$

6. Find the vertex of the equation and identify it as a maximum or a minimum. (62.5, 7812.5); maximum

7. Interpret the vertex of the equation in terms of the situation. If the short side is 62.5 ft and the other side 125 ft, the garden will be the largest possible, with an area of 7812.5 ft^2.

NAME ______________________ DATE __________ PERIOD _____

9-3 Enrichment

Parabolas Through Three Given Points

If you know two points on a straight line, you can find the equation of the line. To find the equation of a parabola, you need three points on the curve.

Here is how to approximate an equation of the parabola through the points (0, −2), (3, 0), and (5, 2).

Use the general equation $y = ax^2 + bx + c$. By substituting the given values for x and y, you get three equations.

(0, −2): $-2 = c$
(3, 0): $0 = 9a + 3b + c$
(5, 2): $2 = 25a + 5b + c$

First, substitute −2 for c in the second and third equations. Then solve those two equations as you would any system of two equations. Multiply the second equation by 5 and the third equation by −3.

$0 = 9a + 3b - 2$ Multiply by 5. $0 = 45a + 15b - 10$
$0 = 25a + 5b - 2$ Multiply by −3. $-6 = -75a - 15b + 6$

$-6 = -30a \qquad - 4$

$a = \frac{1}{15}$

To find b, substitute $\frac{1}{15}$ for a in either the second or third equation.

$0 = 9\left(\frac{1}{15}\right) + 3b - 2$

$b = \frac{7}{15}$

The equation of a parabola through the three points is

$y = \frac{1}{15}x^2 + \frac{7}{15}x - 2.$

Find the equation of a parabola through each set of three points.

1. (1, 5), (0, 6), (2, 3)
   $y = -\frac{1}{2}x^2 - \frac{1}{2}x + 6$
2. (−5, 0), (0, 0), (8, 100)
   $y = \frac{25}{26}x^2 + \frac{125}{26}x$
3. (4, −4), (0, 1), (3, −2)
   $y = -\frac{1}{4}x^2 - \frac{1}{4}x + 1$
4. (1, 3), (6, 0), (0, 0)
   $y = -\frac{3}{5}x^2 + \frac{18}{5}x$
5. (2, 2), (5, −3), (0, −1)
   $y = -\frac{19}{30}x^2 + \frac{83}{30}x - 1$
6. (0, 4), (4, 0), (−4, 4)
   $y = -\frac{1}{8}x^2 - \frac{1}{2}x + 4$

NAME ______________________ DATE ____________ PERIOD _____

9-4 Lesson Reading Guide

Solving Quadratic Equations by Using the Quadratic Formula

Get Ready for the Lesson

Read the introduction to Lesson 9-4 in your textbook.

Your teacher asks you to predict when 17% of the population will consist of people born outside the United States. What equation should you use to make the prediction?

$17 = 0.006t^2 - 0.080t + 5.281$

Read the Lesson

1. Suppose you want to solve $12x^2 + 7x = 15$ using the Quadratic Formula.
   - a. What should you do first?
     Rewrite the equation in the form $ax^2 + bx + c = 0$.
   - b. What are the values you need to substitute for a, b, and c in the Quadratic Formula?
     $a = 12$, $b = 7$, and $c = -15$
   - c. Apply the Quadratic Formula using the above values, but do not solve the equation.
     $$x = \frac{-7 \pm \sqrt{7^2 - 4(12)(-15)}}{2(12)}$$

2. a. You can use the discriminant to determine the number of real roots for a quadratic equation. What is the discriminant?
   The discriminant is the expression under the radical sign in the Quadratic Formula, $b^2 - 4ac$.

   b. Complete the statements below so that each statement is true.

   When the value of the discriminant is **zero**, there is one real root.

   When the value of the discriminant is **positive**, there are two real roots.

   When the value of the discriminant is **negative**, there are no real roots.

Remember What You Learned

3. To help remember the methods for solving a quadratic equation, explain how you would choose the best method for solving a quadratic equation $ax^2 + bx + c = 0$.
   Sample answer: Use graphing for approximate solutions. Use the Quadratic Formula for exact solutions. Use factoring when the factors are easy to determine. Use completing the square when b is an even number.

NAME ______________________ DATE ____________ PERIOD _____

9-4 Study Guide and Intervention

Solving Quadratic Equations by Using the Quadratic Formula

Quadratic Formula To solve the standard form of the quadratic equation, $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

Quadratic Formula	the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ that gives the solutions of $ax^2 + bx + c = 0$, where $a \neq 0$

Example 1 **Solve $x^2 + 2x = 3$ by using the Quadratic Formula.**

Rewrite the equation in standard form.

$x^2 + 2x = 3$ Original equation

$x^2 + 2x - 3 = 3 - 3$ Subtract 3 from each side.

$x^2 + 2x - 3 = 0$ Simplify.

Now let $a = 1$, $b = 2$, and $c = -3$ in the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{16}}{2}$$
$$x = \frac{-2 + 4}{2} \quad \text{or} \quad x = \frac{-2 - 4}{2}$$
$$= 1 \qquad\qquad = -3$$

The solution set is $\{-3, 1\}$.

Example 2 **Solve $x^2 - 6x - 2 = 0$ by using the Quadratic Formula. Round to the nearest tenth if necessary.**

For this equation $a = 1$, $b = -6$, and $c = -2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{6 \pm \sqrt{44}}{2}$$
$$x = \frac{6 + \sqrt{44}}{2} \quad \text{or} \quad x = \frac{6 - \sqrt{44}}{2}$$
$$\approx 6.3 \qquad\qquad \approx -0.3$$

The solution set is $\{-0.3, 6.3\}$.

Exercises

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 3x + 2 = 0$ **1, 2**
2. $m^2 - 8m = -16$ **4**
3. $16r^2 - 8r = -1$ **$\frac{1}{4}$**
4. $x^2 + 5x = 6$ **−6, 1**
5. $3x^2 + 2x = 8$ **$-2, \frac{4}{3}$**
6. $8x^2 - 8x - 5 = 0$ **−0.4, 1.4**
7. $-4c^2 + 19c = 21$ **$\frac{7}{4}, 3$**
8. $2p^2 + 6p = 5$ **−3.7, 0.7**
9. $48x^2 + 22x - 15 = 0$ **$-\frac{5}{6}, \frac{3}{8}$**
10. $8x^2 - 4x = 24$ **$-\frac{3}{2}, 2$**
11. $2p^2 + 5p = 8$ **−3.6, 1.1**
12. $8y^2 + 9y - 4 = 0$ **−1.5, 0.3**
13. $2x^2 + 9x + 4 = 0$ **$-4, -\frac{1}{2}$**
14. $8y^2 + 17y + 2 = 0$ **$-2, -\frac{1}{8}$**
15. $3z^2 + 5z - 2 = 0$ **$-2, \frac{1}{3}$**
16. $-2x^2 + 8x + 4 = 0$ **−0.4, 4.4**
17. $a^2 + 3a = 2$ **−3.6, 0.6**
18. $2y^2 - 6y + 4 = 0$ **1, 2**

Lesson 9-4

NAME ______________________ DATE ____________ PERIOD _____

9-4 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Using the Quadratic Formula

The Discriminant In the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The discriminant can be used to determine the number of real roots for a quadratic equation.

Case 1: $b^2 - 4ac < 0$	Case 2: $b^2 - 4ac = 0$	Case 3: $b^2 - 4ac > 0$
no real roots	one real root	two real roots

Example **State the value of the discriminant for each equation. Then determine the number of real roots.**

a. $12x^2 + 5x = 4$

Write the equation in standard form.

$12x^2 + 5x = 4$ Original equation

$12x^2 + 5x - 4 = 4 - 4$ Subtract 4 from each side.

$12x^2 + 5x - 4 = 0$ Simplify.

Now find the discriminant.

$b^2 - 4ac = (5)^2 - 4(12)(-4)$

$= 217$

Since the discriminant is positive, the equation has two real roots.

b. $2x^2 + 3x = -4$

$2x^2 + 3x = -4$ Original equation

$2x^2 + 3x + 4 = -4 + 4$ Add 4 to each side.

$2x^2 + 3x + 4 = 0$ Simplify.

$b^2 - 4ac = (3)^2 - 4(2)(4)$

$= -23$

Since the discriminant is negative, the equation has no real roots.

Exercises

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

1. $3x^2 + 2x - 3 = 0$ 40, 2 real roots
2. $3n^2 - 7n - 8 = 0$ 145, 2 real roots
3. $2d^2 - 10d - 9 = 0$ 172, 2 real roots
4. $4x^2 = x + 4$ 65, 2 real roots
5. $3x^2 - 13x = 10$ 289, 2 real roots
6. $6x^2 - 10x + 10 = 0$ −140, no real roots
7. $2k^2 - 20 = -k$ 161, 2 real roots
8. $6p^2 = -11p - 40$ −839, no real roots
9. $9 - 18x + 9x^2 = 0$ 0, 1 real root
10. $12x^2 + 9 = -6x$ −396, no real roots
11. $9a^2 = 81$ 2916, 2 real roots
12. $16y^2 + 16y + 4 = 0$ 0, 1 real roots
13. $8x^2 + 9x = 2$ 145, 2 real roots
14. $4a^2 - 4a + 4 = 3$ 0, 1 real root
15. $3b^2 - 18b = -14$ 156, 2 real roots

NAME ______________________ DATE ____________ PERIOD _____

9-4 Skills Practice

Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $u^2 - 49 = 0$ −7, 7
2. $n^2 - n - 20 = 0$ −4, 5
3. $s^2 - 5s - 36 = 0$ −4, 9
4. $b^2 + 11b + 30 = 0$ −6, −5
5. $c^2 - 7c = -3$ 0.5, 6.5
6. $p^2 + 4p = -1$ −3.7, −0.3
7. $a^2 - 9a + 22 = 0$ ∅
8. $x^2 + 6x + 3 = 0$ −5.4, −0.6
9. $2x^2 + 5x - 7 = 0$ $-3\frac{1}{2}$, 1
10. $2h^2 - 3h = -1$ $\frac{1}{2}$, 1
11. $2p^2 + 5p + 4 = 0$ ∅
12. $2g^2 + 7g = 9$ $-4\frac{1}{2}$, 1
13. $3t^2 + 2t - 3 = 0$ −1.4, 0.7
14. $3x^2 - 7x - 6 = 0$ $-\frac{2}{3}$, 3

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

15. $q^2 + 4q + 3 = 0$ 4; 2 real roots
16. $m^2 + 2m + 1 = 0$ 0; 1 real root
17. $a^2 - 4a + 10 = 0$ −24; no real roots
18. $w^2 - 6w + 7 = 0$ 8; 2 real roots
19. $z^2 - 2z - 7 = 0$ 32; 2 real roots
20. $y^2 - 10y + 25 = 0$ 0; 1 real root
21. $2d^2 + 5d - 8 = 0$ 89; 2 real roots
22. $2s^2 + 6s + 12 = 0$ −60; no real roots
23. $2u^2 - 4u + 10 = 0$ −64; no real roots
24. $3h^2 + 7h + 3 = 0$ 13; 2 real roots

NAME ________________ DATE ________ PERIOD ____

9-4 Practice

Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $g^2 + 2g - 3 = 0$ **−3, 1**
2. $a^2 + 8a + 7 = 0$ **−7, −1**
3. $v^2 - 4v + 6 = 0$ **∅**
4. $d^2 - 6d + 7 = 0$ **1.6, 4.4**
5. $2z^2 + 9z - 5 = 0$ **$-5, \frac{1}{2}$**
6. $2r^2 + 12r + 10 = 0$ **−5, −1**
7. $2b^2 - 9b = -12$ **∅**
8. $2h^2 - 5h = 12$ **$-1\frac{1}{2}, 4$**
9. $3p^2 + p = 4$ **$-1\frac{1}{3}, 1$**
10. $3m^2 - 1 = -8m$ **−2.8, 0.1**
11. $4y^2 + 7y = 15$ **$-3, 1\frac{1}{4}$**
12. $1.6n^2 + 2n + 2.5 = 0$ **∅**
13. $4.5k^2 + 4k - 1.5 = 0$ **−1.2, 0.3**
14. $\frac{1}{2}c^2 + 2c + \frac{3}{2} = 0$ **−3, −1**
15. $3w^2 - \frac{3}{4}w = \frac{1}{2}$ **−0.3, 0.6**

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

16. $a^2 + 8a + 16 = 0$ **0; 1 real root**
17. $c^2 + 3c + 12 = 0$ **−39; no real roots**
18. $2w^2 + 12w = -7$ **88; 2 real roots**
19. $2u^2 + 15u = -30$ **−15; no real roots**
20. $4n^2 + 9 = 12n$ **0; 1 real root**
21. $3g^2 - 2g = 3.5$ **46; 2 real roots**
22. $2.5k^2 + 3k - 0.5 = 0$ **14; 2 real roots**
23. $\frac{3}{4}d^2 - 3d = -4$ **−3; no real roots**
24. $\frac{1}{4}s^2 = -s - 1$ **0; 1 real root**

CONSTRUCTION For Exercises 25 and 26, use the following information.

A roofer tosses a piece of roofing tile from a roof onto the ground 30 feet below. He tosses the tile with an initial downward velocity of 10 feet per second.

25. Write an equation to find how long it takes the tile to hit the ground. Use the model for vertical motion, $H = -16t^2 + vt + h$, where H is the height of an object after t seconds, v is the initial velocity, and h is the initial height. (*Hint:* Since the object is thrown down, the initial velocity is negative.) **$H = -16t^2 - 10t + 30$**

26. How long does it take the tile to hit the ground? **about 1.1 s**

27. **PHYSICS** Lupe tosses a ball up to Quyen, waiting at a third-story window, with an initial velocity of 30 feet per second. She releases the ball from a height of 6 feet. The equation $h = -16t^2 + 30t + 6$ represents the height h of the ball after t seconds. If the ball must reach a height of 25 feet for Quyen to catch it, does the ball reach Quyen? Explain. (*Hint:* Substitute 25 for h and use the discriminant.) **No; the discriminant, −316, is negative, so there is no solution.**

NAME ________________ DATE ________ PERIOD ____

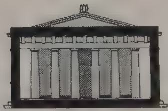
9-4 Word Problem Practice

Solving Quadratic Equations by Using the Quadratic Formula

1. **BUSINESS** Tanya runs a catering business. Based on her records, her weekly profit can be approximated by the function $f(x) = x^2 + 2x - 37$, where x is the number of meals she caters. If $f(x)$ is negative, it means that the business has lost money. What is the least number of meals that Tanya needs to cater in order to have a profit? **6 meals**

2. **AERONAUTICS** At liftoff, the space shuttle *Discovery* has a constant acceleration of 16.4 feet per second squared and an initial velocity of 1341 feet per second due to the rotation of Earth. The distance *Discovery* has traveled t seconds after liftoff is given by the equation $d(t) = 1341t + 8.2t^2$. How long after liftoff has *Discovery* traveled 40,000 feet? Round your answer to the nearest tenth. **25.8 seconds**

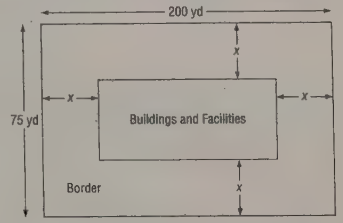
3. **ARCHITECTURE** The Golden Ratio appears in the design of the Greek Parthenon because the width and height of the façade are related by the equation $\frac{W + H}{W} = \frac{W}{H}$. If the height of a model of the Parthenon is 16 inches, what is its width? Round your answer to the nearest tenth. **25.9 in.**



4. **CRAFTS** Madelyn cut a 60-inch pipe cleaner into two unequal pieces, and then she used each piece to make a square. The sum of the areas of the squares was 117 square inches. Let x = the length of one piece. Write and solve an equation to represent the situation and find the lengths of the two original pieces.
$\left(\frac{60 - x}{4}\right)^2 + \left(\frac{x}{4}\right)^2 = 117$
24 in. and 36 in.

SITE DESIGN For Exercises 5–7, refer to the following information. Round your answers to the nearest tenth.

The town of Smallport plans to build a new water treatment plant on a rectangular piece of land 75 yards wide and 200 yards long. The buildings and facilities need to cover an area of 10,000 square yards. The town's zoning board wants the site designer to allow as much room as possible between each edge of the site and the buildings and facilities. Let x = the width of the border.



5. Use an equation similar to area = length × width to represent the situation. **10,000 = (200 − 2x)(75 − 2x)**

6. Write the equation in standard quadratic form and solve for x. **$4x^2 - 550x + 5000 = 0$; 9.8 and 127.7**

7. What should be the width of the border? **9.79 yd**

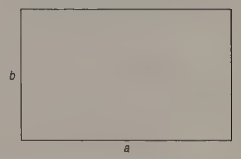
NAME ______________________ DATE __________ PERIOD ____

9-4 Enrichment

Golden Rectangles

A **golden rectangle** has the property that its sides satisfy the following proportion.

$$\frac{a+b}{a} = \frac{a}{b}$$



Two quadratic equations can be written from the proportion. These are sometimes called **golden quadratic** equations.

1. In the proportion, let $a = 1$. Use cross-multiplication to write a quadratic equation.

$b^2 + b - 1 = 0$

2. Solve the equation in Exercise 1 for b.

$b = \frac{-1 + \sqrt{5}}{2}$

3. In the proportion, let $b = 1$. Write a quadratic equation in a.

$a^2 - a - 1 = 0$

4. Solve the equation in Exercise 3 for a.

$a = \frac{1 + \sqrt{5}}{2}$

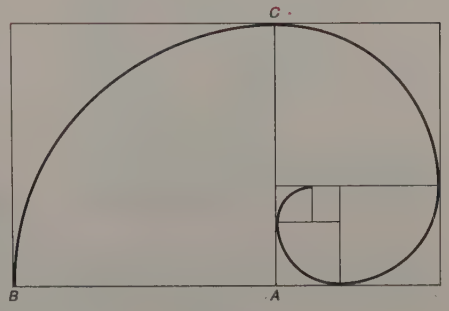
5. Explain why $\frac{1}{2}(\sqrt{5} + 1)$ and $\frac{1}{2}(\sqrt{5} - 1)$ are called golden ratios.

They are the ratios of the sides in a golden rectangle. The first is the ratio of the long side to the short side; the second is short side: long side.

Another property of golden rectangles is that a square drawn inside a golden rectangle creates another, smaller golden rectangle.

In the design at the right, opposite vertices of each square have been connected with quarters of circles.

For example, the arc from point B to point C is created by putting the point of a compass at point A. The radius of the arc is the length BA.



6. On a separate sheet of paper, draw a larger version of the design. Start with a golden rectangle with a long side of 10 inches.

The short side should be about $6\frac{3}{16}$ inches.

NAME ______________________ DATE __________ PERIOD ____

9-5 Lesson Reading Guide

Exponential Functions

Get Ready for the Lesson

Read the introduction to Lesson 9-5 in your textbook.

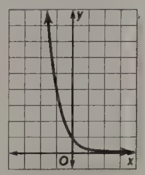
If Mr. Warther had carved a ninth layer of pliers, how many pliers would he have carved?

2^9 or 512 pliers

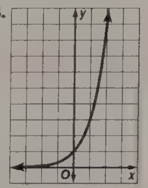
Read the Lesson

1. The graphs of two exponential functions of the form $y = a^x$ are shown below.

A.



B.



a. In Graph A, the value of a is greater than 0 and less than 1. The y values decrease as the x values increase.

b. In Graph B, the value of a is greater than 1. The y values increase as the x values increase.

2. a. When you look for a pattern of exponential behavior in a set of data, what is the pattern you are looking for?

Values of x (domain values) that are at regular intervals and values of y (range values) that have a common factor.

b. If a set of data has a negative common factor, does it display exponential behavior?

No

Remember What You Learned

3. What comparisons can you make between the quadratic function $y = x^2$ and the exponential function $y = 2^x$ to help remember the differences between quadratic and exponential functions?

Sample answer: In the quadratic function, the variable is the base and the exponent is a number; in the exponential function, the variable is the exponent and the base is a number.

Lesson 9-5

Answers

NAME ______________________ DATE ____________ PERIOD _____

9-5 Study Guide and Intervention

Exponential Functions

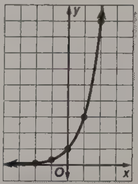
Graph Exponential Functions

Exponential Function	a function defined by an equation of the form $y = a^x$, where $a > 0$ and $a \neq 1$

You can use values of x to find ordered pairs that satisfy an exponential function. Then you can use the ordered pairs to graph the function.

Example 1 **Graph $y = 3^x$. State the y-intercept.**

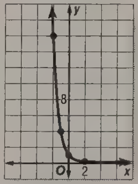
x	y
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9



The y-intercept is 1.

Example 2 **Graph $y = \left(\frac{1}{4}\right)^x$. Use the graph to determine the approximate value of $\left(\frac{1}{4}\right)^{-0.5}$.**

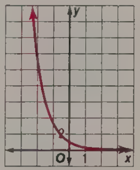
x	y
−2	16
−1	4
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$



The value of $\left(\frac{1}{4}\right)^{-0.5}$ is about 2.

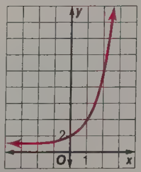
Exercises

1. Graph $y = 0.3^x$. State the y-intercept. Then use the graph to determine the approximate value of $0.3^{-1.5}$. Use a calculator to confirm the value. **1; about 6**

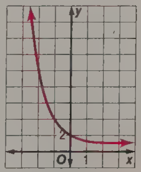


Graph each function. State the y-intercept.

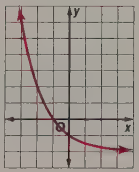
2. $y = 3^x + 1$ **2**



3. $y = \left(\frac{1}{3}\right)^x + 1$ **2**



4. $y = \left(\frac{1}{2}\right)^x - 2$ **−1**



NAME ______________________ DATE ____________ PERIOD _____

9-5 Study Guide and Intervention *(continued)*

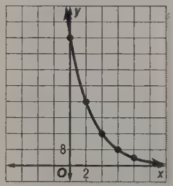
Exponential Functions

Identify Exponential Behavior It is sometimes useful to know if a set of data is exponential. One way to tell is to observe the shape of the graph. Another way is to observe the pattern in the set of data.

Example **Determine whether the set of data displays exponential behavior.**

x	0	2	4	6	8	10
y	64	32	16	8	4	2

Method 1: Graph the Data



The graph shows rapidly decreasing values of y as x increases. This is characteristic of exponential behavior.

Method 2: Look for a Pattern

The domain values increase by regular intervals of 2, while the range values have a common factor of $\frac{1}{2}$. Since the domain values increase by regular intervals and the range values have a common factor, the data are probably exponential.

Exercises

Determine whether the data in each table display exponential behavior. Explain why or why not.

1.

x	0	1	2	3
y	5	10	15	20

No; the domain values are at regular intervals, and the range values have a common difference 5.

2.

x	0	1	2	3
y	3	9	27	81

Yes; the domain values are at regular intervals, and the range values have a common factor 3.

3.

x	−1	1	3	5
y	32	16	8	4

Yes; the domain values are at regular intervals, and the range values have a common factor $\frac{1}{2}$.

4.

x	−1	0	1	2	3
y	3	3	3	3	3

No; the domain values are at regular intervals, but the range values do not change.

5.

x	−5	0	5	10
y	1	0.5	0.25	0.125

Yes; the domain values are at regular intervals, and the range values have a common factor 0.5.

6.

x	0	1	2	3	4
y	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$

Yes; the domain values are at regular intervals, and the range values have a common factor $\frac{1}{3}$.

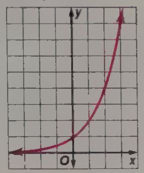
NAME ______________________ DATE ____________ PERIOD _____

9-5 Skills Practice

Exponential Functions

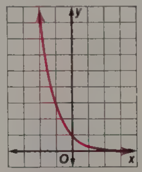
Graph each function. State the y-intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

1. $y = 2^x$; $2^{2.3}$



1; 4.9

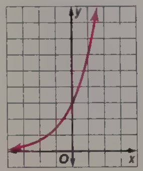
2. $y = \left(\frac{1}{3}\right)^x$; $\left(\frac{1}{3}\right)^{-1.6}$



1; 5.8

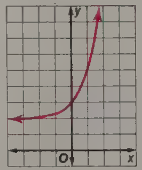
Graph each function. State the y-intercept.

3. $y = 3(2^x)$



3

4. $y = 3^x + 2$



3

Determine whether the data in each table display exponential behavior. Explain why or why not.

5.

x	−3	−2	−1	0
y	9	12	15	18

No; the domain values are at regular intervals and the range values have a common difference 3.

6.

x	0	5	10	15
y	20	10	5	2.5

Yes; the domain values are at regular intervals and the range values have a common factor 0.5.

7.

x	4	8	12	16
y	20	40	80	160

Yes; the domain values are at regular intervals and the range values have a common factor 2.

8.

x	50	30	10	−10
y	90	70	50	30

No; the domain values are at regular intervals and the range values have a common difference 20.

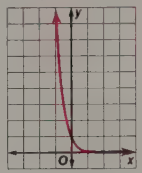
NAME ______________________ DATE ____________ PERIOD _____

9-5 Practice

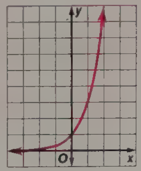
Exponential Functions

Graph each function. State the y-intercept. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

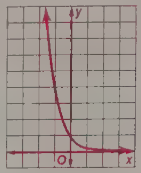
1. $y = \left(\frac{1}{10}\right)^x$; $\left(\frac{1}{10}\right)^{-0.5}$ 1; 3.2



2. $y = 3^x$; $3^{1.9}$ 1; 8.1

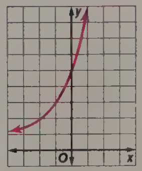


3. $y = \left(\frac{1}{4}\right)^x$; $\left(\frac{1}{4}\right)^{-1.4}$ 1; 7.0

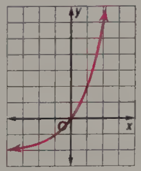


Graph each function. State the y-intercept.

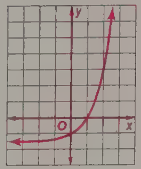
4. $y = 4(2^x) + 1$ 5



5. $y = 2(2^x - 1)$ 0



6. $y = 0.5(3^x - 3)$ −1



Determine whether the data in each table display exponential behavior. Explain why or why not.

7.

x	2	5	8	11
y	480	120	30	7.5

Yes; the domain values are at regular intervals and the range values have a common factor 0.25.

8.

x	21	18	15	12
y	30	23	16	9

No; the domain values are at regular intervals and the range values have a common difference 7.

9. LEARNING Ms. Klemperer told her English class that each week students tend to forget one sixth of the vocabulary words they learned the previous week. Suppose a student learns 60 words. The number of words remembered can be described by the function $W(x) = 60\left(\frac{5}{6}\right)^x$, where x is the number of weeks that pass. How many words will the student remember after 3 weeks? about 35

10. BIOLOGY Suppose a certain cell reproduces itself in four hours. If a lab researcher begins with 50 cells, how many cells will there be after one day, two days, and three days? (*Hint:* Use the exponential function $y = 50(2^x)$.) 3200 cells; 204,800 cells; 13,107,200 cells

NAME ______________________ DATE ____________ PERIOD _____

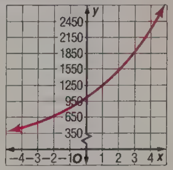
9-5 Word Problem Practice

Exponential Functions

1. **WASTE** Suppose the waste generated by nonrecycled paper and cardboard products is approximated by the following function.

$$y = 1000(2)^{0.3x}$$

Sketch the exponential function on the coordinate grid below.



2. **MONEY** Tatyana's grandfather gave her one penny on the day she was born. He plans to double the amount he gives her every day. Estimate how much she will receive from him on the 12th day of her life. about $20

3. **PICTURE FRAMES** Since a picture frame includes a border, the picture must be smaller in area than the entire frame. The table shows the relationship between picture area and frame length for a particular line of frames. Is this an exponential relationship? Explain. No; there is no common factor between the picture areas.

Side Length (in)	Picture Area (in^2)
5	6
6	12
7	20
8	30
9	42

4. **DEPRECIATION** The value of Royce Company's computer equipment is decreasing in value according to the following function.

$$y = 4000(0.87)^x$$

In the equation, x is the number of years that have elapsed since the equipment was purchased and y is in dollars. What was the value 5 years after it was purchased? Round your answer to the nearest dollar. $1994

METEOROLOGY The atmospheric pressure (in millibars) at a given altitude x, in meters, can be approximated by the following function. The function is valid for values of x between 0 and 10,000.

$$f(x) = 1038(1.000134)^{-x}$$

5. What is the pressure at sea level? 1038 millibars

6. The McDonald Observatory in Texas is at an altitude of 2000 meters. What is the approximate atmospheric pressure there? 794 millibars

7. As altitude increases, what happens to atmospheric pressure? It decreases.

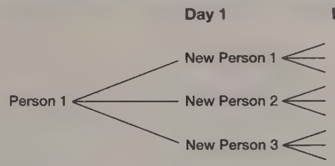
NAME ______________________ DATE ____________ PERIOD _____

9-5 Enrichment

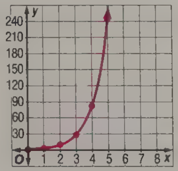
Pay it Forward

The idea behind "pay it forward" is that on the first day, one person does a good deed for three different people. Then, on the second day, those three people will each perform good deeds for 3 more people, so that on Day 2, there are 3 × 3 or 9 good deeds being done. Continue this process to fill in the chart. A tree diagram will help you fill in the chart.

Day	# of Deeds
0	1
1	3
2	9
3	27
4	81
5	243



1. Graph the data you found in the chart as ordered pairs and connect with a smooth curve.



2. What type of function is your graph from problem 1? Write an equation that can be used to determine the number of good deeds on any given day, x. exponential; $y = 3^x$

3. How many good deeds will be performed on Day 21? 10,460,353,203

The formula, $\frac{3^{n+1} - 3}{2}$, can be used to determine the **total** number of good deeds that have been performed, where n represents the day. For example, on Day 2, there have been 3 + 9 or 12 good deeds performed. Using the formula, you get, $\frac{3^{2+1} - 3}{2}$ or $\frac{3^3 - 3}{2}$ + $\frac{27 - 3}{2}$ or a total of 12 good deeds performed.

4. Use this formula to determine the approximate number of good deeds that have been performed through Day 21. 15,690,529,800

5. Look up the world population. How does your number from Exercise 4 compare to the world population? The world population is approximately 6,560,526,046. The number of good deeds performed is greater than the entire world population.

NAME ______________________________ DATE ____________ PERIOD _____

9-5 Spreadsheet Activity

Compound Interest

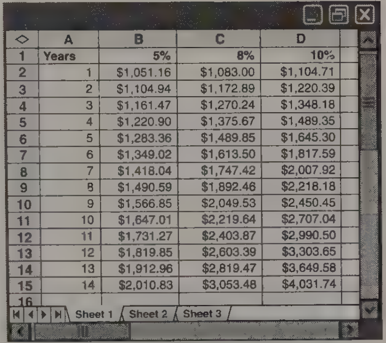
Banks often use spreadsheets to calculate and store financial data. One application is to calculate compound interest on an account.

Example **Use a spreadsheet to find the time it will take an investment of $1000 to double. Suppose you can choose from investments that have annual interest rates of 5%, 8%, or 10% compounded monthly.**

The compound interest equation is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principal or initial investment, A is the final amount of the investment, r is the annual interest rate, n is the number of times interest is paid, or compounded, each year, and t is the number of years. In this case, $P = 1000$ and $n = 12$.

Step 1 Use Column A of the spreadsheet for the years.

Step 2 Columns B, C, and D contain the formulas for the final amounts of the investments. Format the cells in these columns as currency so that the amounts are shown in dollars and cents. Each formula will use the values in Column A as the value of t. For example, in cell C3, the formula is 1000 * (1 + (0.08/12))^(12 * A3).



◇	A	B	C	D
1	Years	5%	8%	10%
2	1	$1,051.16	$1,083.00	$1,104.71
3	2	$1,104.94	$1,172.89	$1,220.39
4	3	$1,161.47	$1,270.24	$1,348.18
5	4	$1,220.90	$1,375.67	$1,489.35
6	5	$1,283.36	$1,489.85	$1,645.30
7	6	$1,349.02	$1,613.50	$1,817.59
8	7	$1,418.04	$1,747.42	$2,007.92
9	8	$1,490.59	$1,892.46	$2,218.18
10	9	$1,566.85	$2,049.53	$2,450.45
11	10	$1,647.01	$2,219.64	$2,707.04
12	11	$1,731.27	$2,403.87	$2,990.50
13	12	$1,819.85	$2,603.39	$3,303.65
14	13	$1,912.96	$2,819.47	$3,649.58
15	14	$2,010.83	$3,053.48	$4,031.74
16				

Study the spreadsheet for the times when each investment exceeds $2000. At 5%, the $1000 will double in 14 years, at 8% it will double in 9 years, and at 10% it will double in 7 years.

Exercises

Use the spreadsheet of accounts involving compound interest.

1. How are the doubling times affected if the accounts compound interest quarterly instead of monthly? **The 5% and 8% accounts double in the same number of years as before, but the 10% account will double in 8 years.**

2. How long will it take each account to reach $4000 if the interest is compounded monthly? quarterly? **monthly: 5% in 28 yr, 8% in 18 yr, 10% in 14 yr; quarterly: 5% in 28 yr, 8% in 18 yr, 10% in 15 yr**

3. How do the interest rate and the number of times the interest is compounded affect the growth of an investment? **The accounts with higher interest rate and interest compounded more often grow more quickly.**

NAME ______________________________ DATE ____________ PERIOD _____

9-6 Lesson Reading Guide

Growth and Decay

Get Ready for the Lesson

Read the introduction to Lesson 9-6 in your textbook.

Suppose you want to predict the restaurant industry sales in the year 2009. What number should be substituted for t? **13**

Read the Lesson

Match an equation to each situation, and then indicate whether the situation is an example of exponential growth or decay.

1. A coin had a value of $1.17 in 1995. Its value has been increasing at a rate of 9% per year. **A; growth**
   - **A.** $y = 1.17(1.09)^t$
   - **B.** $y = 1.17(0.91)^t$

2. A business owner has just paid $6000 for a computer. It depreciates at a rate of 22% per year. How much will it be worth in 5 years? **B; decay**
   - **A.** $A = 6000(1.22)^5$
   - **B.** $A = 6000(0.78)^5$

3. A city had a population of 14,358 residents in 2002. Since then, its population has been decreasing at a rate of about 5.5% per year. **B; decay**
   - **A.** $A = 14{,}358(1.055)^t$
   - **B.** $A = 14{,}358(0.945)^t$

4. Gina deposited $1500 in an account that pays 4% interest compounded quarterly. What will be the worth of the account in 2 years if she makes no deposits and no withdrawals? **B; growth**
   - **A.** $A = 1500(1.02)^2$
   - **B.** $A = 1500(1.01)^8$

Remember What You Learned

5. How can you use what you know about raising a number to the 0 power to help you remember what C represents in the exponential growth equation $A = C(1 + r)^t$ and the exponential decay equation $A = C(1 - r)^t$?

Sample answer: The initial amount is the amount before any time has passed, that is, when $t = 0$. When $t = 0$, $C(1 + r)^t = C(1 + r)^0 = C(1) = C$ and $C(1 - r)^t = C(1 - r)^0 = C(1) = C$. So C represents the initial amount.

NAME ____________________ DATE __________ PERIOD _____

9-6 Study Guide and Intervention

Growth and Decay

Exponential Growth Population increases and growth of monetary investments are examples of **exponential growth**. This means that an initial amount increases at a steady rate over time.

Exponential Growth	The general equation for exponential growth is $y = C(1 + r)^t$. • y represents the final amount. • C represents the initial amount. • r represents the rate of change expressed as a decimal. • t represents time.

Example 1 POPULATION **The population of Johnson City in 2000 was 25,000. Since then, the population has grown at an average rate of 3.2% each year.**

a. Write an equation to represent the population of Johnson City since 2000.

The rate 3.2% can be written as 0.032.

$y = C(1 + r)^t$

$y = 25{,}000(1 + 0.032)^t$

$y = 25{,}000(1.032)^t$

b. According to the equation, what will the population of Johnson City be in the year 2010?

In 2010, t will equal 2010 − 2000 or 10.

Substitute 10 for t in the equation from part a.

$y = 25{,}000(1.032)^{10}$ $t = 10$

$\approx 34{,}256$

In 2010, the population of Johnson City will be about 34,256.

Example 2 INVESTMENT **The Garcias have $12,000 in a savings account. The bank pays 3.5% interest on savings accounts, compounded monthly. Find the balance in 3 years.**

The rate 3.5% can be written as 0.035.

The special equation for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A represents the balance, P is the initial amount, r represents the annual rate expressed as a decimal, n represents the number of times the interest is compounded each year, and t represents the number of years the money is invested.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 12{,}000\left(1 + \frac{0.035}{12}\right)^{36}$

$A \approx 12{,}000(1.00292)^{36}$

$A \approx 13{,}328.09$

In three years, the balance of the account will be $13,326.49.

Exercises

1. **POPULATION** The population of the United States has been increasing at an average annual rate of 0.91%. If the population of the United States was about 297,411,400 in the year 2005, predict the U. S. population in the year 2009. **Source:** U. S. Census Bureau **about 308,385,845**

2. **INVESTMENT** Determine the amount of an investment of $2500 if it is invested at in interest rate of 5.25% compounded monthly for 4 years. **$3082.78**

3. **POPULATION** It is estimated that the population of the world is increasing at an average annual rate of 1.3%. If the population of the world was about 6,472,416,997 in the year 2005, predict the world population in the year 2012. **Source:** U. S. Census Bureau **about 7,084,881,769**

4. **INVESTMENT** Determine the amount of an investment of $100,000 if it is invested at an interest rate of 5.2% compounded quarterly for 12 years. **$185,888.87**

NAME ____________________ DATE __________ PERIOD _____

9-6 Study Guide and Intervention *(continued)*

Growth and Decay

Exponential Decay Radioactive decay and depreciation are examples of **exponential decay**. This means that an initial amount decreases at a steady rate over a period of time.

Exponential Decay	The general equation for exponential decay is $y = C(1 - r)^t$. • y represents the final amount. • C represents the initial amount. • r represents the rate of decay expressed as a decimal. • t represents time.

Example DEPRECIATION **The original price of a tractor was $45,000. The value of the tractor decreases at a steady rate of 12% per year.**

a. Write an equation to represent the value of the tractor since it was purchased.

The rate 12% can be written as 0.12.

$y = C(1 - r)^t$ General equation for exponential decay

$y = 45{,}000(1 - 0.12)^t$ $C = 45{,}000$ and $r = 0.12$

$y = 45{,}000(0.88)^t$ Simplify.

b. What is the value of the tractor in 5 years?

$y = 45{,}000(0.88)^t$ Equation for decay from part a

$y = 45{,}000(0.88)^5$ $t = 5$

$y \approx 23{,}747.94$ Use a calculator.

In 5 years, the tractor will be worth about $23,747.94.

Exercises

1. **POPULATION** The population of Bulgaria has been decreasing at an annual rate of 0.89%. If the population of Bulgaria was about 7,450,349 in the year 2005, predict its population in the year 2015. **Source:** U. S. Census Bureau **about 6,813,204**

2. **DEPRECIATION** Carl Gossell is a machinist. He bought some new machinery for about $125,000. He wants to calculate the value of the machinery over the next 10 years for tax purposes. If the machinery depreciates at the rate of 15% per year, what is the value of the machinery (to the nearest $100) at the end of 10 years? **about $24,600**

3. **ARCHAEOLOGY** The *half-life* of a radioactive element is defined as the time that it takes for one-half a quantity of the element to decay. Radioactive Carbon-14 is found in all living organisms and has a half-life of 5730 years. Consider a living organism with an original concentration of Carbon-14 of 100 grams.

   a. If the organism lived 5730 years ago, what is the concentration of Carbon-14 today? **50 g**

   b. If the organism lived 11,460 years ago, determine the concentration of Carbon-14 today. **25 g**

4. **DEPRECIATION** A new car costs $32,000. It is expected to depreciate 12% each year for 4 years and then depreciate 8% each year thereafter. Find the value of the car in 6 years. **about $16,242.63**

NAME ______________________ DATE __________ PERIOD _____

9-6 Skills Practice

Growth and Decay

POPULATION **For Exercises 1 and 2, use the following information.**

The population of New York City increased from 8,008,278 in 2000 to 8,168,388 in 2005. The annual rate of population increase for the period was about 0.4%. Source: www.nyc.gov

1. Write an equation for the population t years after 2000. $P = 8{,}008{,}278(1.004)^t$

2. Use the equation to predict the population of New York City in 2015. **about 8,502,465**

SAVINGS **For Exercises 3 and 4, use the following information.**

The Fresh and Green Company has a savings plan for its employees. If an employee makes an initial contribution of $1000, the company pays 8% interest compounded quarterly.

3. If an employee participating in the plan withdraws the balance of the account after 5 years, how much will be in the account? **$1485.95**

4. If an employee participating in the plan withdraws the balance of the account after 35 years, how much will be in the account? **$15,996.47**

5. **HOUSING** Mr. and Mrs. Boyce bought a house for $96,000 in 1995. The real estate broker indicated that houses in their area were appreciating at an average annual rate of 4%. If the appreciation remained steady at this rate, what was the value of the Boyce's home in 2005? **about $142,103**

MANUFACTURING **For Exercises 6 and 7, use the following information.**

Zeller Industries bought a piece of weaving equipment for $60,000. It is expected to depreciate at an average rate of 10% per year.

6. Write an equation for the value of the piece of equipment after t years. $E = 60{,}000(0.90)^t$

7. Find the value of the piece of equipment after 6 years. **about $31,886**

8. **FINANCES** Kyle saved $500 from a summer job. He plans to spend 10% of his savings each week on various forms of entertainment. At this rate, how much will Kyle have left after 15 weeks? **$102.95**

9. **TRANSPORTATION** Tiffany's mother bought a car for $9000 five years ago. She wants to sell it to Tiffany based on a 15% annual rate of depreciation. At this rate, how much will Tiffany pay for the car? **about $3993**

NAME ______________________ DATE __________ PERIOD _____

9-6 Practice

Growth and Decay

COMMUNICATIONS **For Exercises 1 and 2, use the following information.**

Sports radio stations numbered 220 in 1996.

1. Write an equation for the number of sports radio stations for t years after 1996. $R = 220(1.143)^t$

2. If the trend continues, predict the number of sports radio stations in this format for the year 2010. **about 1429 stations**

3. **INVESTMENTS** Determine the amount of an investment if $500 is invested at an interest rate of 4.25% compounded quarterly for 12 years. **$830.41**

4. **INVESTMENTS** Determine the amount of an investment if $300 is invested at an interest rate of 6.75% compounded semiannually for 20 years. **$1131.73**

5. **HOUSING** The Greens bought a condominium for $110,000 in 2005. If its value appreciates at an average rate of 6% per year, what will the value be in 2010? **about $147,205**

DEFORESTATION **For Exercises 6 and 7, use the following information.**

During the 1990s, the forested area of Guatemala decreased at an average rate of 1.7%.

6. If the forested area in Guatemala in 1990 was about 34,400 square kilometers, write an equation for the forested area for t years after 1990. $C = 34{,}400(0.983)^t$

7. If this trend continues, predict the forested area in 2015. **about 22,407.65 km^2**

8. **BUSINESS** A piece of machinery valued at $25,000 depreciates at a steady rate of 10% yearly. What will the value of the piece of machinery be after 7 years? **about $11,957**

9. **TRANSPORTATION** A new car costs $18,000. It is expected to depreciate at an average rate of 12% per year. Find the value of the car in 8 years. **about $6473**

10. **POPULATION** The population of Osaka, Japan declined at an average annual rate of 0.05% for the five years between 1995 and 2000. If the population of Osaka was 11,013,000 in 2000 and it continues to decline at the same rate, predict the population in 2050. **about 10,741,000**

NAME ______________________ DATE __________ PERIOD _____

9-6 Word Problem Practice

Growth and Decay

1. **DEPRECIATION** The value of a new plasma television depreciates by about 7% each year. Aeryn purchases a 50-inch plasma television for $5000. What is its value after 4 years? Round your answer to the nearest hundred. **about $3700**

2. **MONEY** Hans opens a savings account by depositing $1200 in an account that earns 3 percent interest compounded weekly. How much will his investment be worth in 10 years? Assume that there are exactly 52 weeks in a year and round your answer to the nearest cent. **$1619.69**

3. **HIGHER EDUCATION** The table lists the average costs of attending a four-year college in the United States during the 2005–2006 school years.

College Sector	Tuition and Fees	Room and Board
Four-year Public	$5,491	$6,636
Four-year Private	$21,235	$7,791

Source: www.collegeboard.com

Russ's parents invested money in a savings account earning an average of 4.5 percent interest, compounded monthly. After 15 years, they have exactly the right amount to cover the tuition, fees, room and board for Russ's first year at a public college. What was their initial investment? Round your answer to the nearest dollar. **$6182**

4. **POPULATION** In the 2000 U.S. Census, the population of the United States was estimated at 281 million. The annual rate of growth is about 1.1%. At this rate, what is the expected population at the time of the 2020 census? Round your answer to the nearest ten million. **350 million**

MEDICINE For Exercises 5–7, use the following information. If necessary, round your answers to the nearest tenth.

When doctors prescribe medication, they have to consider the rate at which the body filters a drug from the bloodstream. Suppose it takes the human body 6 days to filter out half of the Flu-B-Gone vaccine. The amount of Flu-B-Gone vaccine remaining in the bloodstream x hours after an injection is given by the equation $y = y_0(0.5)^{\frac{x}{6}}$, where y_0 is the initial amount. Suppose a doctor injects a patient with 20 μg (micrograms) of Flu-B-Gone.

5. How much of the vaccine will remain after 1 day? **17.8 μg**

6. How much of the vaccine will remain after 12 days? **5 μg**

7. After how many days will the amount of vaccine be less than 1 μg? **after 26 days**

NAME ______________________ DATE __________ PERIOD _____

9-6 Enrichment

Growth and Decay

Sierpinski Triangle is an example of a fractal that changes exponentially. Start with an equilateral triangle and find the midpoints of each side. Then connect the midpoints to form a smaller triangle. Remove this smaller triangle from the larger one.

Repeat the process to create the next triangle in the sequence. Find the midpoints of the sides of the three remaining triangles and connect them to form smaller triangles to be removed.

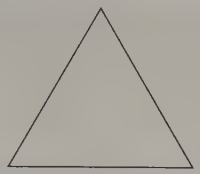


figure 1

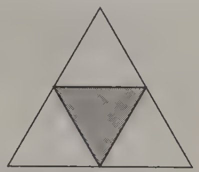


figure 2

Cut out: $\frac{1}{4}$

Area = $1 - \frac{1}{4}$ or $\frac{3}{4}$

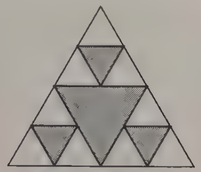


figure 3

Cut out: $\frac{1}{4} + \frac{3}{16}$ or $\frac{7}{16}$

Area = $1 - \frac{7}{16}$ or $\frac{9}{16}$

1. Find the next triangle in the sequence. How much has been cut out? What is the area of the fourth figure in the sequence? $\frac{1}{4} + \frac{3}{16} + \frac{9}{64}$ or $\frac{37}{64}$; $\frac{1}{4}$ $1 - \frac{37}{64}$ or $\frac{27}{64}$

2. Make a conjecture as to what you need to multiply the previous amount cut by to find the new amount cut. $\frac{3}{4}$

3. Fill in the chart to represent the amount cut and the area remaining for each triangle in the sequence.

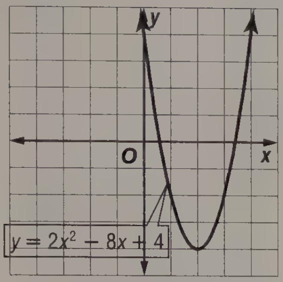
figure	1	2	3	4	5	6
amount cut	0	$\frac{1}{4}$	$\frac{7}{16}$	$\frac{37}{64}$	$\frac{175}{256}$	$\frac{781}{1024}$
area remaining	1	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{27}{64}$	$\frac{81}{256}$	$\frac{243}{1024}$

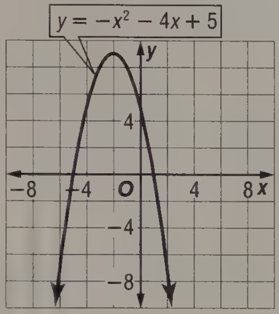
4. Write an equation to represent the area that is left in the nth triangle in the sequence. $\left(\frac{3}{4}\right)^n$

5. If this process is continued, make a conjecture as to the remaining area. **The remaining area gets very close to 0.**

Chapter 9 Assessment Answer Key

Quiz 1 (Lessons 9-1 and 9-2)
Page 51

1.


2.


$x = -2$; $(-2, 9)$; maximum

3. C

4. −4, 6

5. $0 < m < 1$, $1 < m < 2$

Quiz 2 (Lesson 9-3)
Page 51

1. 1.5, 8.5

2. −16, 2

3. $\frac{81}{4}$

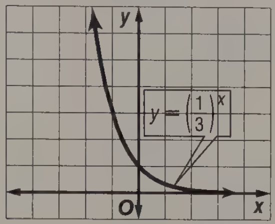
4. 2, −10

5. 3.4, −2.7

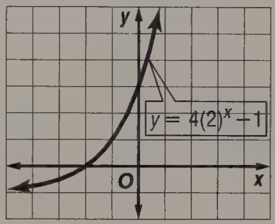
Quiz 3 (Lessons 9-4 and 9-5)
Page 52

1. −1, $\frac{7}{4}$

2. −167; no real roots



3. 1; 6.5



4. 3

Quiz 4 (Lesson 9-6)
Page 52

1. Sample answer: $y = 1500\left(1 + \frac{.0485}{4}\right)^{4 \times 12}$

2. $2,675.04

3. Sample answer: $y = 35{,}000(0.92)^t$

4. $19,524.63

5. ≈ 867,154 people

Mid-Chapter Test
Page 53

1. D

2. F

3. B

4. H

5. B

6. −1, 8

7. $0 < x < 1$, $4 < x < 5$

8. −5.2, 1.7

9. −6.9, 2.9

10. height is 2.4 in.; base is 6.4 in.

Chapter 9 Assessment Answer Key

Vocabulary Test
Page 54

1. zeros
2. parabola
3. roots
4. general equation for exponential growth
5. quadratic formula
6. vertex
7. general equation for exponential decay
8. quadratic function
9. quadratic equation
10. exponential function
11. Sample answer: The axis of symmetry of a parabola is the line that divides a parabola into two matching halves.
12. Sample answer: The discriminant is the expression, $b^2 - 4ac$, that is under the radical sign in the quadratic formula.

Form 1
Page 55

1. C
2. G
3. C
4. F
5. D
6. J
7. B
8. J
9. A
10. G

Page 56

11. B
12. F
13. C
14. J
15. A
16. H
17. B
18. H
19. A
20. H

B: 1 real root

Chapter 9 Assessment Answer Key

Form 2A
Page 57

1. B
2. G
3. D
4. H
5. A
6. G
7. D
8. H
9. D

Page 58

10. J
11. A
12. H
13. A
14. J
15. A
16. G
17. A
18. G
19. D
20. H

B: $x = 1$

Form 2B
Page 59

1. A
2. G
3. C
4. J
5. A
6. J
7. B
8. G
9. A

Page 60

10. H
11. D
12. F
13. B
14. J
15. C
16. J
17. B
18. J
19. C
20. H

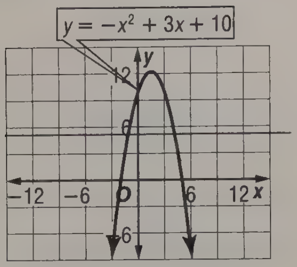
B: −16, 16

Chapter 9 Assessment Answer Key

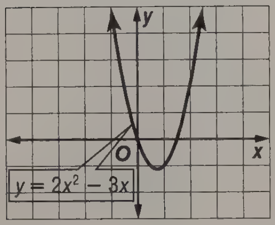
Form 2C

Page 61

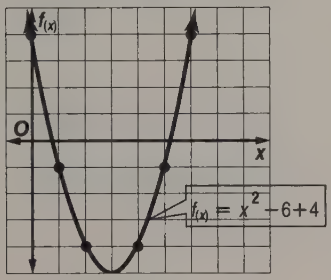
1.



2.



3. $x = 2$; $(2, -5)$; minimum



4. $0 < x < 1, 5 < x < 6$

5. −2.2, 4.2

6. −23, 3

7. −4, 2.5

8. 0.8, 11.2

9. $-\frac{1}{3}, \frac{3}{5}$

10. −14.6, −1.4

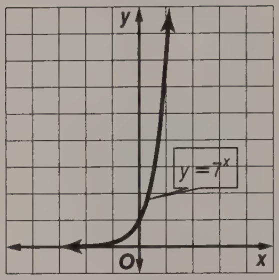
Page 62

11. 148; 2 real roots

12. 0; 1 real root

13. length is 8.8 in.; width is 3.8 in.

14. about 201 lb



15. 1

16. 3.2

17. No; the domain values are at regular intervals, but the range values do not have a positive common factor.

18. Yes; the domain values at regular intervals and the range values have a common factor 3.

19. \$1434.67

20. about \$14,797.81

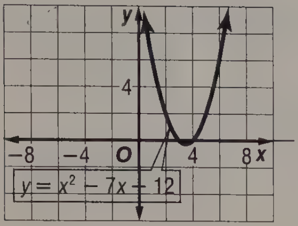
B: about −1.2 and −0.1

Chapter 9 Assessment Answer Key

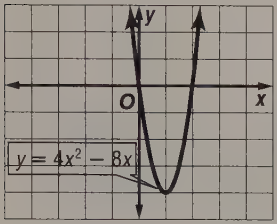
Form 2D

Page 63

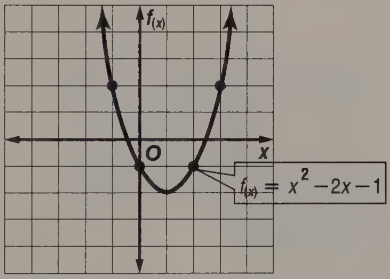
1.



2.



3. $x = 1$; $(1, -3)$; maximum



4. $-1 < x < 0$, $2 < x < 3$

5. −1, 19

6. −8.9, −1.1

7. $-6, \frac{3}{2}$

8. −14.2, −0.8

9. $-\frac{3}{4}, \frac{2}{3}$

10. 1.8, 12.2

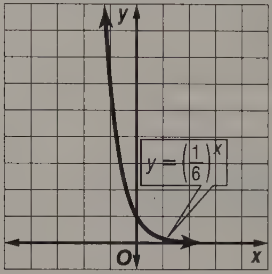
Page 64

11. −56; no real roots

12. 0; 1 real root

13. length is 8.7 in.; width is 5.7 in.

14. about 185 lb



15. 1

16. 3.5

17. No; the domain values are at regular intervals, but the range values have a common difference 3.

18. Yes; the domain values are at regular intervals and the range values have a common factor 2.

19. about $5030.49

20. about $4595.27

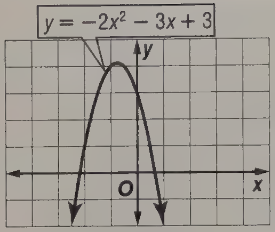
B: $\frac{1}{2}$

Chapter 9 Assessment Answer Key

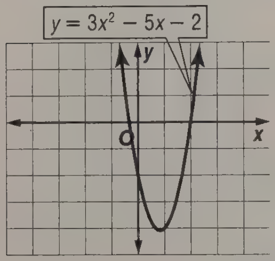
Form 3

Page 65

1.

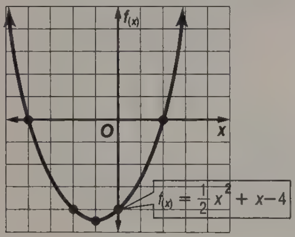


2.

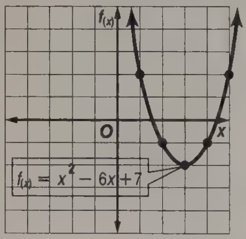


3. $x = \frac{5}{2}$; $\left(\frac{5}{2}, -\frac{109}{4}\right)$; minimum

4. $x = 4$



5. $-4, 2$



6. $1 < x < 2, 4 < x < 5$

7. $-24, -2$

8. 10.3, 23.7

9. $-0.6, 2.9$

10. $-5 \pm \sqrt{25 - c}$

Page 66

11. $-\frac{2}{3}, \frac{5}{7}$ or $-0.7, 0.7$

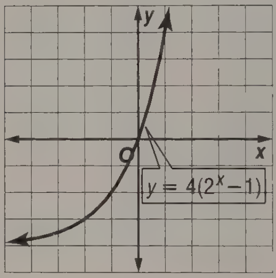
12. $-0.6, 3.1$

13. 0 times

14. -18 or 18

15. 7 and 17 or -17 and -7

16. 4



17. 0

18. Yes; the domain values are at regular intervals and the range values have a common factor of 2.

19. \$112,206.07

20. 5 years

B: $\frac{-7 \pm \sqrt{49 + 16a}}{2a}$

Chapter 9 Assessment Answer Key

Page 67, Extended-Response Test Scoring Rubric

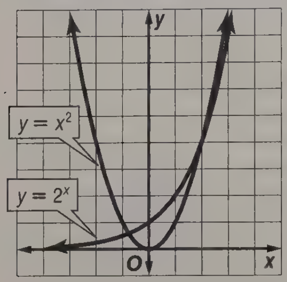
Score	General Description	Specific Criteria
4	**Superior** A correct solution that is supported by well-developed, accurate explanations	• Shows thorough understanding of the concepts of *graphing quadratic and exponential functions, solving quadratic equations, using the discriminant, exponential growth, and exponential decay.* • Uses appropriate strategies to solve problems. • Computations are correct. • Written explanations are exemplary. • Graphs are accurate and appropriate. • Goes beyond requirements of some or all problems.
3	**Satisfactory** A generally correct solution, but may contain minor flaws in reasoning or computation	• Shows an understanding of the concepts of *graphing quadratic and exponential functions, solving quadratic equations, using the discriminant, exponential growth, and exponential decay.* • Uses appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are effective. • Graphs are mostly accurate and appropriate. • Satisfies all requirements of problems.
2	**Nearly Satisfactory** A partially correct interpretation and/or solution to the problem	• Shows an understanding of most of the concepts of *graphing quadratic and exponential functions, solving quadratic equations, using the discriminant, exponential growth, and exponential decay.* • May not use appropriate strategies to solve problems. • Computations are mostly correct. • Written explanations are satisfactory. • Graphs are mostly accurate. • Satisfies the requirements of most of the problems.
1	**Nearly Unsatisfactory** A correct solution with no supporting evidence or explanation	• Final computation is correct. • No written explanations or work is shown to substantiate the final computation. • Graphs may be accurate but lack detail or explanation. • Satisfies minimal requirements of some of the problems.
0	**Unsatisfactory** An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given	• Shows little or no understanding of most of the concepts of *graphing quadratic and exponential functions, solving quadratic equations, using the discriminant, exponential growth, and exponential decay.* • Does not use appropriate strategies to solve problems. • Computations are incorrect. • Written explanations are unsatisfactory. • Graphs are inaccurate or inappropriate. • Does not satisfy requirements of problems. • No answer may be given.

Chapter 9 Assessment Answer Key

Page 67, Extended-Response Test
Sample Answers

In addition to the scoring rubric found on page A29, the following sample answers may be used as guidance in evaluating extended-response assessment items.

1. Sample answer: $f(x) = x^2$; $f(x) = 2^x$



The domains of the two functions are the same, all real numbers. The range of $f(x) = x^2$ is $y \geq 0$, while the range of $f(x) = 2^x$ is $y > 0$.
The quadratic function has a vertical line of symmetry, $x = 0$, and is symmetrical about that line. However, the exponential function has no symmetry.

2a. Sample answer: $x^2 + x + 1 = 0$; -3; The student should explain that the discriminant is the expression under the radical sign in the Quadratic Formula. Since there is no real square root of a negative number, there are no real roots of an equation whose discriminant is negative.

2b. Sample answer: $x^2 + 2x + 1 = 0$; 0; The student should explain that zero has only one square root. Thus, when the determinant is zero the Quadratic Formula yields only one value when you simplify the numerator of the formula.

2c. Sample answer: $x^2 + x - 1 = 0$; Since this quadratic equation does not factor and the roots are not integers, completing the square would be the better method to use to solve the equation.

3a. Sample answer: 5%; about 14.2 years

3b. Sample answer: 10%; about 6.6 years

3c. Sample answer: about 10.4 years; about \$8320

Chapter 9 Assessment Answer Key

Standardized Test Practice

Page 68

1. Ⓐ ● Ⓒ Ⓓ

2. ● Ⓖ Ⓗ Ⓙ

3. ● Ⓑ Ⓒ Ⓓ

4. Ⓕ Ⓖ ● Ⓙ

5. Ⓐ Ⓑ Ⓒ ●

6. Ⓕ Ⓖ ● Ⓙ

7. Ⓐ Ⓑ ● Ⓓ

8. Ⓕ ● Ⓗ Ⓙ

9. Ⓐ Ⓑ Ⓒ ●

10. Ⓕ Ⓖ Ⓗ ●

Page 69

11. ● Ⓑ Ⓒ Ⓓ

12. Ⓕ ● Ⓗ Ⓙ

13. Ⓐ Ⓑ Ⓒ ●

14. Ⓕ ● Ⓗ Ⓙ

15. Ⓐ ● Ⓒ Ⓓ

16. ● Ⓖ Ⓗ Ⓙ

17. ● Ⓑ Ⓒ Ⓓ

18.

			5	.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	①		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	●		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

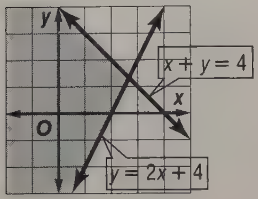
19.

			1	.			
⓪	⓪	⓪	⓪		⓪	⓪	⓪
①	①	①	●		①	①	①
②	②	②	②		②	②	②
③	③	③	③		③	③	③
④	④	④	④		④	④	④
⑤	⑤	⑤	⑤		⑤	⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨	⑨

Standardized Test Practice
Page 70

20. $\{y \mid y \geq 5\}$

21.



22. $3x^3 + 2x^2 + 4x - 3$

23. $-4x^2y + 9xy - 8y^2$

24. $9a^4 - 4$

25. $(x + 7)(x + 5)$

26. $(2m + 5)(m + 3)$

27. $\{-12, 12\}$

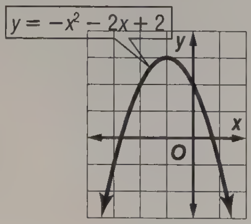
28. 10 s and about 5.6 s

29. about 9,209,514 people

30a. $x = -1$

30b. $(-1, 3)$; maximum

30c.



Chapter 9 Assessment Answer Key

Unit 3 Test

Page 71

1. $20x^4y^6$

2. $512x^{18}y^{18}$

3. $\frac{b^6}{144g^6}$

4. $6 + 5xy^2 + 3x^2y - 7x^4$

5. 3

6. $3a + 2b - 4c$

7. 2

8. $p^2 + 2p - 15$

9. $a^3 - a^2 - 13a - 14$

10. $25t^2 + 10ts + s^2$

11. $9k^2 - 1$

12. $30mnp^2$

13. $4pq(p - 4 + 2q)$

14. $(m - 2)(m - 9)$

15. $(3a - 4)(a - 3)$

16. $3(3r - 2s)(3r + 2s)$

17. $2p - 1$

Page 72

18. $\left\{-\frac{1}{2}, 0\right\}$

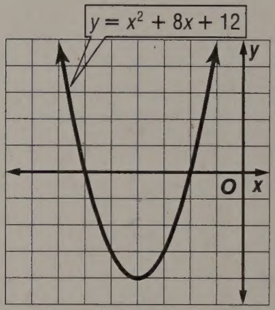
19. $\{4, 9\}$

20. $\left\{-\frac{2}{5}, 7\right\}$

21. $\left\{-\frac{1}{2}, 0, \frac{1}{2}\right\}$

22. $\{-9\}$

23. $x = -4; (-4, -4)$



24. $\{-6, 8\}$

25. -8

26. $\{-1.8, 1.1\}$

27. No; the domain intervals are at regular intervals and the range values have a common difference 2.

28. about 850,966 people

29. about \$61,032

30. \$1531.51

Algebra 1

Contents

Unit 1 Foundations for Functions
  1 The Language and Tools of Algebra
  2 Solving Linear Equations
Unit 2 Linear Functions
  3 Functions and Patterns
  4 Analyzing Linear Equations
  5 Solving Systems of Linear Equations
  6 Solving Linear Inequalities
Unit 3 Polynomials and Nonlinear Functions
  7 Polynomials
  8 Factoring
  9 Quadratic and Exponential Functions
Unit 4 Advanced Expressions and Data Analysis
  10 Radical Expressions and Triangles
  11 Rational Expressions and Equations
  12 Statistics and Probability

Math Online Visit us online at **glencoe.com**

Mc Graw Hill Glencoe

The McGraw·Hill Companies

ISBN-13: 978-0-07-873952-1
ISBN-10: 0-07-873952-7
90000
9 780078 739521
www.glencoe.com

P8-BTR-455